MHT CET 2023 : 13th May Morning Shift

Mathematics

Question 1

If $f(x)=x^3+bx^2+cx+d$ **and** $0<b^2<c$**, then in** $(-\infty,\infty)$

Options:

A. $f(x)$ has a local maxima.

B. $f(x)$ is strictly increasing function.

C. $f(x)$ is bounded.

D. $f(x)$ is strictly decreasing function.

Answer: B

Solution:

$f(x)=x^3+bx^2+cx+d$

$\therefore \quad f'(x)=3x^2+2bx+c$

Now its discriminant $=4\left(b^2-3c\right)$

$\Rightarrow \quad 4\left(b^2-c\right)-8c<0,$ as $b^2<c$ and $c>0$

$\Rightarrow \quad f'(x)>0$ for all $x\in R$

$\Rightarrow$ f is strictly increasing on R.

Question 2

Differentiation of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ **w.r.t.** $\cos^{-1}\left(\sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}}\right)$ **is**

Options:

A. $\frac{1}{2}$

B. 1

C. 2

D. $\frac{1}{4}$

Answer: B

Solution:

Let $u=\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$

and $\mathrm{v}=\cos^{-1}\left[\sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}}\right]$

Put $x=\tan\theta$, then $\theta=\tan^{-1}x$

$$\therefore \mathrm{u}=\tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]=\tan^{-1}\left[\frac{\sec\theta-1}{\tan\theta}\right]$$

$$=\tan^{-1}\left[\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right]=\tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$$

$$=\tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right]=\tan^{-1}\left(\tan\frac{\theta}{2}\right)=\frac{\theta}{2}$$

$$\therefore \quad \mathrm{u}=\frac{\tan^{-1}x}{2} \quad \text{.... (i)}$$

$$\mathrm{v}=\cos^{-1}\left[\sqrt{\frac{1+\sqrt{1+\tan^2\theta}}{2\sqrt{1+\tan^2\theta}}}\right]$$

$$=\cos^{-1}\left[\sqrt{\frac{1+\sec\theta}{2\sec\theta}}\right]$$

$$=\cos^{-1}\left[\sqrt{\frac{1+\frac{1}{\cos\theta}}{\frac{2}{\cos\theta}}}\right]$$

$$=\cos^{-1}\left[\sqrt{\frac{1+\cos\theta}{2}}\right]$$

$$=\cos^{-1}\left(\sqrt{\frac{2\cos^2\left(\frac{\theta}{2}\right)}{2}}\right)$$

$$=\cos^{-1}\left(\cos\frac{\theta}{2}\right)=\frac{\theta}{2}$$

$$\therefore \quad \mathrm{v}=\frac{\tan^{-1}x}{2} \quad \text{.... (ii)}$$

From (i) and (ii), we get $\mathrm{u}=\mathrm{v}$

$\therefore \quad \frac{\mathrm{du}}{\mathrm{dv}}=1$

Question 3

The function $\mathrm{f}(\mathrm{t})=\frac{1}{\mathrm{t}^2+\mathrm{t}-2}$ where $\mathrm{t}=\frac{1}{x-1}$ is discontinuous at

Options:

A. $-2, 1$

B. $2, \frac{1}{2}$

C. $\frac{1}{2}, 1$

D. $2, 1$

Answer: B

Solution:

$f(t) = \frac{1}{t^2+t-2} = \frac{1}{(t+2)(t-1)}$

$f(t)$ is not defined at $t = -2$ and $t = 1$.

$$t = -2$$
$$\Rightarrow \frac{1}{x-1} = -2$$
$$\Rightarrow x = \frac{1}{2}$$
$$t = 1$$
$$\Rightarrow \frac{1}{x-1} = 1$$
$$\Rightarrow x = 2$$

$\therefore$ The function $f(t)$ is discontinuous at $x = \frac{1}{2}$ and $x = 2$.

Question 4

A random variable X has the following probability distribution

$X = x$	0	1	2
$P(X = x)$	$4k - 10k^2$	$5k - 1$	$3k^3$

then $P(X < 2)$ is

Options:

A. $\frac{2}{9}$

B. $\frac{5}{9}$

C. $\frac{8}{9}$

D. $\frac{4}{9}$

Answer: C

Solution:

$$\text{Since } \sum_{x=0}^{2} \mathrm{P}(\mathrm{X}=x)=1$$

$$4\mathrm{k}-10\mathrm{k}^2+5\mathrm{k}-1+3\mathrm{k}^3=1$$
$$\Rightarrow 3\mathrm{k}^3-10\mathrm{k}^2+9\mathrm{k}-2=0$$
$$\Rightarrow (\mathrm{k}-1)(\mathrm{k}-2)(3\mathrm{k}-1)=0$$
$$\Rightarrow \mathrm{k}=1 \text{ or } \mathrm{k}=2 \text{ or } \mathrm{k}=\frac{1}{3}$$

For $\mathrm{k}=1$ or $\mathrm{k}=2$,

$\mathrm{P}(\mathrm{X}=0)<0$, which is not possible

$\therefore \quad \mathrm{k}=\frac{1}{3}$

$$\text{Now,} \quad \mathrm{P}(\mathrm{X}<2)=P(X=0)+P(X=1)$$
$$=4k-10k^2+5k-1$$
$$=9k-10k^2-1$$
$$=9\left(\frac{1}{3}\right)-10\left(\frac{1}{9}\right)-1$$
$$=\frac{8}{9}$$

Question 5

If $\mathrm{I}=\int \frac{2x-7}{\sqrt{3x-2}}\,\mathrm{d}x$, then I is given by

Options:

A. $\frac{106}{27}(3x-2)^{\frac{3}{2}}+\mathrm{c}$, where c is a constant of integration.

B. $\frac{98}{27}(3x-2)^{\frac{3}{2}}+\mathrm{c}$, where c is a constant of integration.

C. $\frac{4}{27}(3x-2)^{\frac{3}{2}}-\frac{34}{9}(3x-2)^{\frac{1}{2}}+$ c, where c is a constant of integration.

D. $\frac{4}{27}(3x-2)^{\frac{3}{2}}+\frac{34}{9}(3x-2)^{\frac{1}{2}}+$ c, where c is a constant of integration

Answer: C

Solution:

$$\begin{aligned}
\mathrm{I} &= \int \frac{2x-7}{\sqrt{3x-2}}\,\mathrm{d}x \\
&= \int \frac{\frac{2}{3}(3x-2)-\frac{17}{3}}{\sqrt{3x-2}}\,\mathrm{d}x \\
&= \frac{2}{3}\int (3x-2)^{\frac{1}{2}}\,\mathrm{d}x - \frac{17}{3}\int (3x-2)^{\frac{-1}{2}}\,\mathrm{d}x \\
&= \frac{2}{3}\times\frac{(3x-2)^{\frac{3}{2}}}{\frac{3}{2}}\times\frac{1}{3}-\frac{17}{3}\times\frac{(3x-2)^{\frac{1}{2}}}{\frac{1}{2}}\times\frac{1}{3}+\mathrm{c} \\
&= \frac{4}{27}(3x-2)^{\frac{3}{2}}-\frac{34}{9}(3x-2)^{\frac{1}{2}}+\mathrm{c}
\end{aligned}$$

Question 6

Let X be random variable having Binomial distribution $B(7,p)$. If $P[X=3]=5P[X=4]$, then variance of X is

Options:

A. $\frac{7}{6}$

B. $\frac{35}{36}$

C. $\frac{77}{36}$

D. $\frac{1}{36}$

Answer: B

Solution:

$P(X = 3) = 5P(X = 4)$
$\Rightarrow {}^7C_3 p^3 q^4 = 5\,{}^7C_4 p^4 q^3$
$\Rightarrow 5p = q$
$\Rightarrow 5p = 1 - p$
$\Rightarrow 6p = 1$
$\Rightarrow p = \frac{1}{6}$
$\Rightarrow q = 1 - \frac{1}{6} = \frac{5}{6}$
$\text{Variance } = npq$
$= 7 \times \frac{1}{6} \times \frac{5}{6}$
$= \frac{35}{36}$

Question 7

The scalar product of vectors $\overline{a} = \hat{i} + 2\hat{j} + \hat{k}$ and a unit vector along the sum of vectors $\bar{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\bar{c} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}$ is one, then the value of λ is

Options:

A. 1

B. -2

C. -3

D. 2

Answer: C

Solution:

$\overline{\mathrm{b}}+\overline{\mathrm{c}}=(2+\lambda)\hat{\mathrm{i}}-2\hat{\mathrm{j}}+2\hat{\mathrm{k}}$

$$\text{Unit vector } = \frac{\overline{\mathrm{b}}+\overline{\mathrm{c}}}{\sqrt{\overline{\mathrm{b}}+\overline{\mathrm{c}}\,|}}$$

$$= \frac{(2+\lambda)\hat{\mathrm{i}}-2\hat{\mathrm{j}}+2\hat{\mathrm{k}}}{\sqrt{(2+\lambda)^2+(-2)^2+2^2}}$$

$$= \frac{(2+\lambda)\hat{\mathrm{i}}-2\hat{\mathrm{j}}+2\hat{\mathrm{k}}}{\sqrt{\lambda^2+4\lambda+12}}$$

According to the given condition,

$$(\hat{i}+2\hat{j}+\hat{k})\cdot\left(\frac{(2+\lambda)\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{\lambda^2+4\lambda+12}}\right)=1$$

$$\Rightarrow \frac{(2+\lambda)-4+2}{\sqrt{\lambda^2+4\lambda+12}}=1$$

$\Rightarrow \lambda=\sqrt{\lambda^2+4\lambda+12}$

$\Rightarrow \lambda^2=\lambda^2+4\lambda+12$

$\Rightarrow 4\lambda=-12$

$\Rightarrow \lambda=-3$

Question 8

$$\int \frac{\log\left(x^2+a^2\right)}{x^2} dx =$$

Options:

A. $\frac{-\log\left(x^2+\mathrm{a}^2\right)}{x}+\frac{1}{\mathrm{a}}\tan^{-1}\frac{x}{\mathrm{a}}+\mathrm{c}$, where c is a constant of integration.

B. $\frac{-\log\left(x^2+\mathrm{a}^2\right)}{x}+\frac{2}{\mathrm{a}}\tan^{-1}\frac{x}{\mathrm{a}}+\mathrm{c}$, where c is a constant of integration.

C. $\frac{\log\left(x^2+\mathrm{a}^2\right)}{x^2}-\frac{1}{\mathrm{a}}\tan^{-1}\frac{x}{\mathrm{a}}+\mathrm{c}$, where c is a constant of integration.

D. $\frac{\log\left(x^2+\mathrm{a}^2\right)}{x^2}-\frac{2}{\mathrm{a}}\tan^{-1}\frac{x}{\mathrm{a}}+\mathrm{c}$, where c is a constant of integration.

Answer: B

Solution:

$$\text{Let I} = \int \frac{\log\left(x^2+a^2\right)}{x^2}\, dx$$
$$= \int \log\left(x^2+a^2\right) \cdot x^{-2}\, dx$$
$$= \log\left(x^2+a^2\right) \int x^{-2}\, dx$$
$$- \int \left\{\frac{d}{dx}\left[\log\left(x^2+a^2\right)\right] \int x^{-2}\, dx\right\} dx$$
$$= \log\left(x^2+a^2\right) \cdot \left(-\frac{1}{x}\right) - \int \frac{2x}{x^2+a^2} \cdot \left(-\frac{1}{x}\right) dx$$
$$= -\frac{\log\left(x^2+a^2\right)}{x} + 2\int \frac{1}{x^2+a^2}\, dx$$
$$= -\frac{\log\left(x^2+a^2\right)}{x} + \frac{2}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

Question 9

The parametric equations of the curve $x^2 + y^2 + ax + by = 0$ are

Options:

A. $x = \frac{a}{2} + \sqrt{\frac{a^2+b^2}{4}} \cos\theta, y = \frac{b}{2} + \sqrt{\frac{a^2+b^2}{4}} \sin\theta$

B. $x = \frac{a}{2} - \sqrt{\frac{a^2+b^2}{4}} \cos\theta, y = \frac{b}{2} - \sqrt{\frac{a^2+b^2}{4}} \sin\theta$

C. $x = -\frac{a}{2} + \sqrt{\frac{a^2+b^2}{4}} \cos\theta, y = -\frac{b}{2} + \sqrt{\frac{a^2+b^2}{4}} \sin\theta$

D. $x = -\frac{a}{2} - \sqrt{\frac{a^2+b^2}{4}} \cos\theta, y = -\frac{b}{2} - \sqrt{\frac{a^2+b^2}{4}} \sin\theta$

Answer: C

Solution:

$$x^2 + y^2 + ax + by = 0$$
$$\Rightarrow \left(x + \frac{a}{2}\right)^2 + \left(y + \frac{b}{2}\right)^2 = \frac{a^2+b^2}{4}$$

Comparing with $(x-h)^2 + (y-k)^2 = r^2$, we get

$h=-\frac{a}{2}, k=-\frac{b}{2}, r=\sqrt{\frac{a^2+b^2}{4}}$

∴ The parametric equations are

$x=-\frac{a}{2}+\sqrt{\frac{a^2+b^2}{4}}\cos\theta, y=-\frac{b}{2}+\sqrt{\frac{a^2+b^2}{4}}\sin\theta$

Question 10

The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one, then the sides of the triangle (in units) are

Options:

A. 3, 4, 5

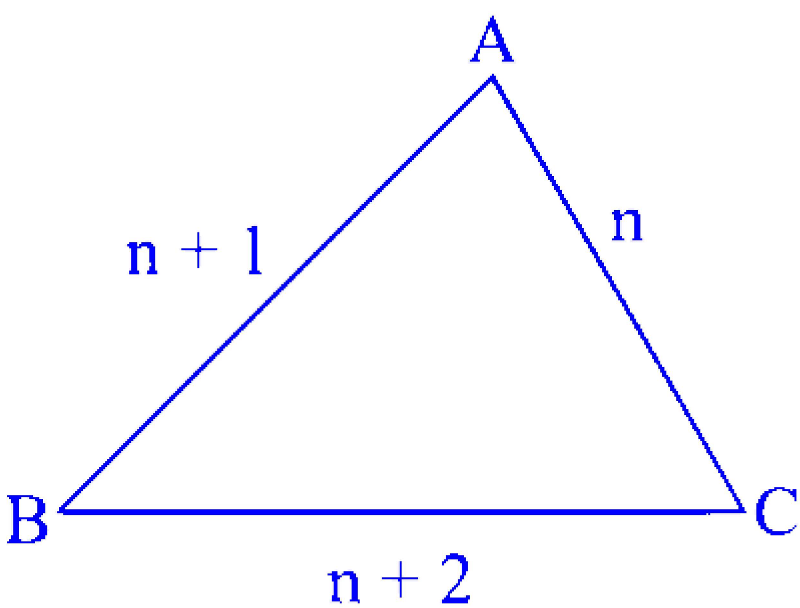
B. 4, 5, 6

C. 5, 6, 7

D. 2, 3, 4

Answer: B

Solution:



Let AC = n, AB = n + 1, BC = n + 2

$\therefore$ Largest angle is A and smallest angle is B.

$\therefore \quad A = 2B$

Since $A + B + C = 180^\circ$

$\therefore \quad 3B + C = 180^\circ$

$\Rightarrow \quad C = 180^\circ - 3B$

$\Rightarrow \quad \sin C = \sin(180^\circ - 3B) = \sin 3B$

By sine rule,

$$\frac{\sin A}{n+2} = \frac{\sin B}{n} = \frac{\sin C}{n+1}$$

$$\frac{\sin 2B}{n+2} = \frac{\sin B}{n} = \frac{\sin 3B}{n+1}$$

$$\Rightarrow \frac{2\sin B \cos B}{n+2} = \frac{\sin B}{n} = \frac{3\sin B - 4\sin^3 B}{n+1}$$

$$\frac{2\cos B}{n+2} = \frac{1}{n} = \frac{3 - 4\sin^2 B}{n+1}$$

$$\therefore \quad \cos B = \frac{n+2}{2n}, 3 - 4\sin^2 B = \frac{n+1}{n}$$

$$\therefore \quad 3 - 4\left(1 - \cos^2 B\right) = \frac{n+1}{n}$$

$$\therefore \quad 3 - 4 + 4\left(\frac{n+2}{2n}\right)^2 = \frac{n+1}{n}$$

$$\Rightarrow -1 + \frac{n^2 + 4n + 4}{n^2} = \frac{n+1}{n}$$

$$\Rightarrow -n^2 + n^2 + 4n + 4 = n^2 + n$$

$$\Rightarrow n^2 - 3n - 4 = 0$$

$$\Rightarrow (n+1)(n-4) = 0$$

$$\Rightarrow n = -1 \text{ or } n = 4$$

But n cannot be negative.

$\therefore$ n $= 4$

$\therefore$ The sides of the Δ are $4, 5, 6$.

Question 11

x, y, z are in G.P. and $\tan^{-1} x, \tan^{-1} y, \tan^{-1} z$ are in A.P., then

Options:

A. $6x = 4y = 3z$

B. $2x = 3y = 6z$

C. $6x = 3y = 2z$

D. $x = y =$ z

Answer: D

Solution:

$x, y,$ z are in G.P.

$\Rightarrow y^2 = xz$ .... (i)

Also, $\tan^{-1} x, \tan^{-1} y, \tan^{-1} z$ are in A.P.

$$\Rightarrow 2\tan^{-1} y = \tan^{-1} x + \tan^{-1} z$$
$$\Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$
$$\Rightarrow \frac{2y}{1-y^2} = \frac{x+z}{1-xz}$$
$$\Rightarrow \frac{2y}{1-xz} = \frac{x+z}{1-xz} \quad \text{...[From (i)]}$$

$\Rightarrow 2y = x +$ z

$\Rightarrow x, y,$ z are in A.P. ..... (ii)

From (i) and (ii), we get

$x = y =$ z

Question 12

In $\triangle ABC$ with usual notation, $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ and $a = \frac{1}{\sqrt{6}}$, then the area of triangle is _______ sq. units.

Options:

A. $\frac{1}{8}$

B. $\frac{1}{24\sqrt{3}}$

C. $\frac{1}{24}$

D. $\frac{1}{8\sqrt{3}}$

Answer: D

Solution:

If $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$, then the triangle is equilateral.

$$\begin{aligned} \therefore \quad \mathrm{A}(\triangle \mathrm{ABC}) &= \frac{\sqrt{3}}{4}\mathrm{a}^2 \\ &= \frac{\sqrt{3}}{4}\left(\frac{1}{\sqrt{6}}\right)^2 \\ &= \frac{\sqrt{3}}{24} = \frac{1}{8\sqrt{3}} \text{ sq. units} \end{aligned}$$

Question 13

If $\hat{\mathrm{a}}$ and $\hat{\mathrm{b}}$ are unit vectors and $\overline{\mathrm{c}} = \hat{\mathrm{b}} - (\hat{\mathrm{a}} \times \overline{\mathrm{c}})$, then minimum value of $[\hat{a}\hat{b}\overline{c}]$ is

Options:

A. 0

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. 1

Answer: A

Solution:

[Note: The question cannot be solved due to insufficient data.]

Question 14

Angles of a triangle are in the ratio $4:1:1$. Then the ratio of its greatest side to its perimeter is

Options:

A. $3:(2+\sqrt{3})$

B. $\sqrt{3}:(2+\sqrt{3})$

C. $\sqrt{3}:(2-\sqrt{3})$

D. $1:(2+\sqrt{3})$

Answer: B

Solution:

Let the angles of the triangle be $4x$, x and x.

$\therefore 4x+x+x=180^\circ \Rightarrow 6x=180^\circ \Rightarrow x=30^\circ$

By sine rule,

$$\frac{\sin 120^\circ}{\mathrm{a}}=\frac{\sin 30^\circ}{\mathrm{b}}=\frac{\sin 30^\circ}{\mathrm{c}}$$

$$\therefore \quad \mathrm{a}:(\mathrm{a}+\mathrm{b}+\mathrm{c})$$

$$=(\sin 120^\circ):(\sin 120^\circ+\sin 30^\circ+\sin 30^\circ)$$

$$=\frac{\sqrt{3}}{2}:\frac{\sqrt{3}+2}{2}=\sqrt{3}:\sqrt{3}+2$$

Question 15

If a continuous random variable X has probability density function $\mathrm{f}(x)$ given by

$$f(x)=\begin{cases} ax & \text{, if } 0 \leq x < 1 \\ a & \text{, if } 1 \leq x < 2 \\ 3a - ax & \text{, if } 2 \leq x \leq 3\text{'} \\ 0 & \text{, otherwise} \end{cases}$$

then a has the value

Options:

A. $\frac{1}{5}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. 1

Answer: C

Solution:

Since $\mathrm{f}(x)$ is the p.d.f. of X,

$$\int_{-\infty}^{\infty} \mathrm{f}(x) \mathrm{d}x = 1$$

$$\Rightarrow \int_0^1 \mathrm{ax}\,\mathrm{d}x + \int_1^2 \mathrm{a}\,\mathrm{d}x + \int_2^3 (3\mathrm{a} - \mathrm{ax})\mathrm{d}x = 1$$

$$\Rightarrow \mathrm{a}\left[\frac{x^2}{2}\right]_0^1 + \mathrm{a}[x]_1^2 + \left[3\mathrm{ax} - \frac{\mathrm{a}x^2}{2}\right]_2^3 = 1$$

$$\Rightarrow a\left(\frac{1}{2}\right) + a(1) + \left(\frac{9a}{2} - 4a\right) = 1$$

$$\Rightarrow 2a = 1$$

$$\Rightarrow a = \frac{1}{2}$$

Question 16

The value of $\cos(18^\circ - A) \cdot \cos(18^\circ + A) - \cos(72^\circ - A)\cos(72^\circ + A)$ is

Options:

A. $\cos 72^\circ$

B. $\sin 54^\circ$

C. $\sin 18^\circ$

D. $\cos 54^\circ$

Answer: B

Solution:

$$\begin{aligned}
&\cos(18^\circ - A)\cos(18^\circ + A) \\
&\quad - \cos(72^\circ - A)\cos(72^\circ + A) \\
&= \cos(18^\circ - A)\cos[90^\circ - (72^\circ - A)] \\
&\quad - \cos(72^\circ - A)\cos[90^\circ - (18^\circ - A)] \\
&= \sin(72^\circ - A)\cos(18^\circ - A) \\
&\quad - \cos(72^\circ - A)\sin(18^\circ - A) \\
&= \sin[(72^\circ - A) - (18^\circ - A)] \\
&= \sin 54^\circ
\end{aligned}$$

Question 17

If $\int x^5 e^{-4x^3}\, dx = \frac{1}{48} e^{-4x^3} f(x) + c$**, where c is a constant of integration, then** $f(x)$ **is given by**

Options:

A. $4x^3 + 1$

B. $-4x^3 - 1$

C. $-2x^3 - 1$

D. $-2x^3 + 1$

Answer: B

Solution:

$$\text{Let I} = \int x^5 e^{-4x^3}\, dx$$

$$\text{Put } x^3 = t \Rightarrow 3x^2\, dx = dt$$

$$\therefore \quad I = \frac{1}{3}\int te^{-4t}dt = \frac{1}{3}\left(t \cdot \frac{e^{-4t}}{-4} - \int 1 \cdot \frac{e^{-4t}}{-4}dt\right)$$

$$= \frac{1}{3}\left(\frac{-te^{-4t}}{4} + \frac{1}{4} \cdot \frac{e^{-4t}}{-4}\right) + c$$

$$= \frac{1}{48}e^{-4t}(-4t - 1) + c$$

$$= \frac{1}{48}e^{-4x^3}\left(-4x^3 - 1\right) + c$$

$$\therefore \quad f(x) = -4x^3 - 1$$

Question 18

The solution of the differential equation $e^{-x}(y+1)dy + \left(\cos^2 x - \sin 2x\right)y\, dx = 0$ at $x = 0, y = 1$ is

Options:

A. $(y+1) + e^x \cos^2 x = 2$

B. $y + \log y = e^x \cos^2 x$

C. $\log(y+1) + e^x \cos^2 x = 1$

D. $y + \log y + e^x \cos^2 x = 2$

Answer: D

Solution:

$$e^{-x}(y+1)dy+\left(\cos^2 x-\sin 2x\right)ydx=0$$
$$\Rightarrow \frac{y+1}{y}dy+e^{x}\left(\cos^2 x-\sin 2x\right)dx=0$$

Integrating on both sides, we get

$$\int\left(1+\frac{1}{y}\right)\mathrm{d}y+\int \mathrm{e}^{x}\left(\cos^2 x-2\sin x\cos x\right)\mathrm{d}x=\mathrm{c}$$
$$\Rightarrow y+\log|y|+\mathrm{e}^{x}\cos^2 x=\mathrm{c}$$
At $x=0, y=1$
$$\therefore \quad 1+\log 1+\mathrm{e}^{0}\cos^2 0=\mathrm{c}$$
$$\Rightarrow \mathrm{c}=2$$
$$\therefore \quad y+\log|y|+\mathrm{e}^{x}\cos^2 x=2$$

Question 19

If $A=\begin{bmatrix}1 & 2 & 3\\ -1 & 1 & 2\\ 1 & 2 & 4\end{bmatrix}$ and A_{ij} is a cofactor of a_{ij} then the value of $a_{21}A_{21}+a_{22}A_{22}+a_{23}A_{23}$ is

Options:

A. 0

B. -2

C. 4

D. 3

Answer: D

Solution:

$$a_{21} = -1, a_{22} = 1, a_{23} = 2$$
$$A_{21} = (-1)^{2+1}\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = (-1)(2) = -2$$
$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = 1(1) = 1$$
$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = (-1)(0) = 0$$
$$\therefore \quad a_{21} A_{21} + a_{22} A_{22} + a_{23} A_{23}$$
$$= (-1)(-2) + 1(1) + 2(0) = 3$$

Question 20

Rate of increase of bacteria in a culture is proportional to the number of bacteria present at that instant and it is found that the number doubles in 6 hours. The number of bacteria becomes _________ times at the end of 18 hours.

Options:

A. 9

B. 6

C. 8

D. 3

Answer: C

Solution:

Let P_0 be the initial population and let the population after t years be P. Then,

$$\frac{dP}{dt} = kP, \text{ where } k > 0$$
$$\Rightarrow \frac{dP}{P} = kdt$$

Integrating on both sides, we get

$$\log P = kt + c$$

When $t = 0, P = P_0$

$\therefore \quad \log P_0 = 0 + c$

$\therefore \quad c = \log P_0$

$$\log P = kt + \log P_0$$

$\Rightarrow \log \frac{P}{P_0} = kt \quad .... (i)$

When $t = 6$ hrs, $P = 2P_0$

$\therefore \quad \log \frac{2P_0}{P_0} = 6k$

$\Rightarrow k = \frac{\log 2}{6}$

$\therefore \quad \log \frac{P}{P_0} = \frac{\log 2}{6} t \quad \ldots$ [From (i)]

When t = 18 hrs, we have

$$\log \frac{P}{P_0} = \frac{\log 2}{6} \times 18$$
$$= 3 \log 2$$

$$\therefore \quad \log \frac{P}{P_0} = \log 8$$
$$\Rightarrow P = 8P_0$$

Question 21

The slope of the normal to the curve $x = \sqrt{t}$ and $y = t - \frac{1}{\sqrt{t}}$ at $t = 4$ is

Options:

A. $\frac{-17}{4}$

B. $\frac{4}{17}$

C. $\frac{-4}{17}$

D. $\frac{17}{4}$

Answer: C

Solution:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1 + \frac{1}{2t^{\frac{3}{2}}}}{\frac{1}{2\sqrt{t}}} = \frac{2t^{\frac{3}{2}} + 1}{t}$$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{t=4} = \frac{2(4)^{\frac{3}{2}} + 1}{4} = \frac{16 + 1}{4} = \frac{17}{4}$$

$\therefore$ Slope of normal at $t = 4$ is $-\frac{1}{\left(\frac{dy}{dx}\right)_{t=4}} = -\frac{4}{17}$

Question 22

If $3f(x) - f\left(\frac{1}{x}\right) = 8\log_2 x^3, x > 0$, then $f(2), f(4), f(8)$ are in

Options:

A. A.P.

B. G.P.

C. H.P.

D. Arithmetico Geometric Progression

Answer: A

Solution:

$$3f(x) - f\left(\frac{1}{x}\right) = 8\log_2 x^3 \quad \text{... (i)}$$

$$\Rightarrow 3f\left(\frac{1}{x}\right) - f(x) = 8\log_2\left(\frac{1}{x}\right)^3 \quad \text{... (ii)}$$

From (i) and (ii), we get

$$8f(x) = 24\log_2 x^3 + 8\log_2\left(\frac{1}{x}\right)^3$$

$$\Rightarrow 8f(x) = 72\log_2 x - 24\log_2 x$$

$$\Rightarrow 8f(x) = 48\log_2 x$$

$$\Rightarrow f(x) = 6\log_2 x$$

$$\therefore \quad f(2) = 6\log_2 2 = 6$$

$$f(4) = 6\log_2 4 = 12$$

$$f(8) = 6\log_2 8 = 18$$

$\therefore \quad f(2), f(4), f(8)$ are in A.P.

Question 23

If the angle between the lines given by $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0; \lambda \geq 0$ is $\tan^{-1}\left(\frac{1}{3}\right)$, then the value of λ is

Options:

A. 1

B. 2

C. $\frac{9}{4}$

D. -1

Answer: B

Solution:

Given equation of pair of lines is

$x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0$

Here, $\mathrm{a} = 1,\ \mathrm{b} = \lambda, \mathrm{c} = 2, \mathrm{f} = \frac{-5}{2},\ \mathrm{g} = \frac{3}{2},\ \mathrm{h} = \frac{-3}{2}$

$$\theta = \tan^{-1}\left(\frac{1}{3}\right) \Rightarrow \tan\theta = \frac{1}{3}$$

$$\text{Since } \tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$$

$$\Rightarrow \frac{1}{3} = \left|\frac{2\sqrt{\left(\frac{-3}{2}\right)^2 - \lambda}}{\lambda + 1}\right|$$

$$\Rightarrow (\lambda + 1)^2 = 9(9 - 4\lambda) \Rightarrow \lambda^2 + 38\lambda - 80 = 0$$
$$\Rightarrow (\lambda + 40)(\lambda - 2) = 0 \Rightarrow \lambda = 2 \quad \ldots[\because \lambda \geq 0]$$

Question 24

A line drawn from the point $\mathrm{A}(1,3,2)$ parallel to the line $\frac{x}{2}=\frac{y}{4}=\frac{z}{1}$, intersects the plane $3x+y+2z=5$ in point B, then co-ordinates of point B are

Options:

A. $\left(\frac{1}{6},\frac{4}{3},\frac{19}{12}\right)$

B. $\left(-\frac{1}{6},-\frac{4}{3},\frac{19}{12}\right)$

C. $\left(\frac{1}{6},\frac{4}{3},-\frac{19}{12}\right)$

D. $\left(-\frac{1}{6},-\frac{4}{3},-\frac{19}{12}\right)$

Answer: A

Solution:

The d.r.s. of the line $\frac{x}{2}=\frac{y}{4}=\frac{z}{1}$ are $2,4,1$.

$\therefore$ The d.r.s. of any line parallel to it are also $2,4,1$.

The equation of the line passing through $\mathrm{A}(1,3,2)$ is $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-2}{1}=\lambda$ (say)

Then, any point on the line is

$B\equiv(2\lambda+1,4\lambda+3,\lambda+2)$

The point B lies on the plane $3x+y+2z=5$.

$$\therefore\quad 3(2\lambda+1)+4\lambda+3+2(\lambda+2)=5$$
$$\Rightarrow 12\lambda+10=5$$
$$\Rightarrow \lambda=\frac{-5}{12}$$
$$\therefore\quad B\equiv\left(\frac{1}{6},\frac{4}{3},\frac{19}{12}\right)$$

Question 25

The value of $\frac{\mathrm{i}^{248}+\mathrm{i}^{246}+\mathrm{i}^{244}+\mathrm{i}^{242}+\mathrm{i}^{240}}{\mathrm{i}^{249}+\mathrm{i}^{247}+\mathrm{i}^{245}+\mathrm{i}^{243}+\mathrm{i}^{241}}$, $(\mathrm{i}=\sqrt{-1})$ is

Options:

A. i

B. 1

C. -1

D. $-$i

Answer: D

Solution:

$$\frac{i^{248}+i^{246}+i^{244}+i^{242}+i^{240}}{i^{249}+i^{247}+i^{245}+i^{243}+i^{241}}$$
$$=\frac{i^{240}\left(i^{8}+i^{6}+i^{4}+i^{2}+1\right)}{i^{241}\left(i^{8}+i^{6}+i^{4}+i^{2}+1\right)}$$
$$=\frac{i^{240}}{i^{241}}$$
$$=\frac{1}{i}$$
$$=\frac{i}{i^{2}}=-i$$

Question 26

If $\mathrm{f}(x)=\int \frac{x^{2}\ \mathrm{d}x}{\left(1+x^{2}\right)\left(1+\sqrt{1+x^{2}}\right)}$ **and** $\mathrm{f}(0)=0$**, then** $\mathrm{f}(1)$ **is**

Options:

A. $\log(1+\sqrt{2})$

B. $\log(1+\sqrt{2})-\frac{\pi}{4}$

C. $\log(1+\sqrt{2})+\frac{\pi}{4}$

D. $\log(1-\sqrt{2})$

Answer: B

Solution:

Put $x=\tan\theta \Rightarrow dx=\sec^2\theta d\theta$

$$\therefore \quad f(x)=\int \frac{\tan^2\theta \sec^2\theta d\theta}{\sec^2\theta(1+\sec\theta)}$$

$$=\int \frac{\tan^2\theta d\theta}{1+\sec\theta}$$

$$=\int \frac{\sin^2\theta d\theta}{\cos\theta(1+\cos\theta)}$$

$$=\int \frac{1-\cos^2\theta d\theta}{\cos\theta(1+\cos\theta)}$$

$$=\int \frac{(1-\cos\theta)d\theta}{\cos\theta}$$

$$=\int \sec\theta d\theta-\int d\theta$$

$$=\log|\sec\theta+\tan\theta|-\theta+c$$

$$\therefore \quad f(x)=\log\left|x+\sqrt{1+x^2}\right|-\tan^{-1}x+c$$

$$\therefore \quad f(0)=\log|0+\sqrt{1+0}|-\tan^{-1}(0)+c$$

$$\Rightarrow \quad 0=\log 1-0+c \Rightarrow c=0$$

$$\therefore \quad f(x)=\log\left|x+\sqrt{1+x^2}\right|-\tan^{-1}x$$

$$\therefore \quad f(1)=\log\left|1+\sqrt{1+1^2}\right|-\tan^{-1}(1)$$

$$=\log(1+\sqrt{2})-\frac{\pi}{4}$$

Question 27

A line L_1 passes through the point, whose p. v. (position vector) $3\hat{i}$, is parallel to the vector $-\hat{i}+\hat{j}+\hat{k}$. Another line L_2 passes through the point having p.v. $\hat{i}+\hat{j}$ is parallel to vector $\hat{i}+\hat{k}$, then the point of intersection of lines L_1 and L_2 has p.v.

Options:

A. $2\hat{i}+2\hat{j}+\hat{k}$

B. $2\hat{i}+\hat{j}+\hat{k}$

C. $2\hat{i}-\hat{j}-\hat{k}$

D. $2\hat{i}-2\hat{j}+\hat{k}$

Answer: B

Solution:

Equation of line L_1 is $\overline{r} = 3\hat{i} + \lambda(-\hat{i} + \hat{j} + \hat{k})$ .... (i)

Equation of line L_2 is $\overline{r'} = \hat{i} + \hat{j} + \lambda'(\hat{i} + \hat{k})$

The point of intersection of L_1 and L_2 will satisfy $\overline{r} = \overline{r'}$

$\Rightarrow 3\hat{i} + \lambda(-\hat{i} + \hat{j} + \hat{k}) = \hat{i} + \hat{j} + \lambda'(\hat{i} + \hat{k})$
$\Rightarrow (3 - \lambda)\hat{i} + \lambda\hat{j} + \lambda\hat{k} = (1 + \lambda')\hat{i} + \hat{j} + \lambda'\hat{k}$
$\Rightarrow 3 - \lambda = 1 + \lambda'$ and $\lambda = 1$
$\Rightarrow \lambda = 1$ and $\lambda' = 1$

Substituting the value of λ in (i), we get the point of intersection.

$\therefore$ The point of intersection of lines L_1 and L_2 has p.v. $2\hat{i} + \hat{j} + \hat{k}$.

Question 28

If $y = \tan^{-1}\left(\frac{4\sin 2x}{\cos 2x - 6\sin^2 x}\right)$, then $\left(\frac{dy}{dx}\right)$ at $x = 0$ is

Options:

A. 3

B. 5

C. 8

D. 1

Answer: C

Solution:

$$y=\tan^{-1}\left(\frac{4\sin 2x}{\cos 2x-6\sin^2 x}\right)$$
$$=\tan^{-1}\left[\frac{4(2\sin x\cos x)}{\left(\cos^2 x-\sin^2 x\right)-6\sin^2 x}\right]$$
$$=\tan^{-1}\left(\frac{8\sin x\cos x}{\cos^2 x-7\sin^2 x}\right)$$
$$=\tan^{-1}\left(\frac{8\tan x}{1-7\tan^2 x}\right)$$
$$=\tan^{-1}\left(\frac{7\tan x+\tan x}{1-7\tan x\cdot\tan x}\right)$$
$$=\tan^{-1}(7\tan x)+\tan^{-1}(\tan x)$$
$$\therefore\quad y=\tan^{-1}(7\tan x)+x$$
$$\therefore\quad \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+(7\tan x)^2}\cdot 7\sec^2 x+1$$
$$=\frac{7\sec^2 x}{1+49\tan^2 x}+1$$
$$\therefore\quad \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=0}\quad \frac{7\sec^2 0}{1+49\tan^2 0}+1$$
$$=\frac{7}{1+0}+1$$
$$=8$$

Question 29

The expression $(p\wedge\sim q)\vee q\vee(\sim p\wedge q)$ is equivalent to

Options:

A. $\sim$ p $\vee$ q

B. $p\wedge q$

C. p $\vee$ q

D. $p\vee\sim q$

Answer: C

Solution:

$(p\wedge\sim q)\vee q\vee(\sim p\wedge q)$
$\equiv[(p\vee q)\wedge(\sim q\vee q)]\vee(\sim p\wedge q)$ ...[Distributive law]

$\equiv [(p \vee q) \wedge T] \vee (\sim p \wedge q) \quad \ldots$ [Complement law]
$\equiv (p \vee q) \vee (\sim p \wedge q)$
....[Identity law]

$\equiv (p \vee q \vee \sim p) \wedge (p \vee q \vee q)$ .... [Distributive law]

$\equiv (T \vee q) \wedge (p \vee q)$ ...[Complement law and Idempotent law]

$\equiv T \wedge (p \vee q) \quad \ldots$ [Identity law]
$\equiv p \vee q \quad \ldots$ [Identity law]

Question 30

The raw data $x_1, x_2, \ldots \ldots, x_n$ is an A.P. with common difference d and first term $0, \bar{x}$ and σ^2 are mean and variance of $x_{\mathrm{i}}, \mathrm{i} = 1, 2, \ldots \ldots \mathrm{n}$, then σ^2 is

Options:

A. $\frac{(n^2+1)d^2}{24}$

B. $\frac{(\mathrm{n}^2-1)\mathrm{d}^2}{24}$

C. $\frac{(\mathrm{n}^2+1)\mathrm{d}^2}{12}$

D. $\frac{(\mathrm{n}^2-1)\mathrm{d}^2}{12}$

Answer: D

Solution:

$$\begin{aligned}
\bar{x} &= \frac{x_1 + x_2 + \ldots + x_n}{n} \\
&= \frac{\frac{n}{2}[2x_1 + (n-1)d]}{n} \\
&= \frac{(n-1)d}{2} \quad \ldots [\because x_1 = 0] \\
\sum x_i^2 &= x_1^2 + x_2^2 + \ldots + x_n^2 \\
&= 0 + d^2 + (2\,d)^2 + \ldots + [(n-1)d]^2 \\
&= d^2\left[1 + 2^2 + 3^2 + \ldots + (n-1)^2\right] \\
&= d^2\left[\frac{n(n-1)(2n-1)}{6}\right] \\
\therefore \quad \sigma^2 &= \frac{1}{n}\sum x_i^2 - (\bar{x})^2 \\
&= \frac{d^2(n-1)(2n-1)}{6} - \left[\frac{(n-1)d}{2}\right]^2 \\
&= \frac{d^2(n-1)}{2}\left(\frac{2n-1}{3} - \frac{n-1}{2}\right) \\
&= \frac{d^2(n-1)}{2}\left(\frac{n+1}{6}\right) = \frac{(n^2-1)d^2}{12}
\end{aligned}$$

Question 31

The particular solution of differential equation $e^{\frac{dy}{dx}} = (x+1), y(0) = 3$ **is**

Options:

A. $y = x \log x - x + 2$

B. $y = (x+1)\log(x+1) - x + 3$

C. $y = (x+1)\log(x+1) + x - 3$

D. $y = x \log x + x - 2$

Answer: B

Solution:

$e^{\frac{dy}{dx}} = (x+1)$

$\Rightarrow \frac{dy}{dx} = \log(x+1)$

Integrating on both sides, we get

$$\int dy = \int \log(x+1)dx + c$$

$$\Rightarrow y = x\log(x+1) - \int \frac{x}{x+1}dx + c$$

$$= x\log(x+1) - \int \frac{x+1-1}{x+1}dx + c$$

$$= x\log(x+1) - \int \left(1 - \frac{1}{x+1}\right)dx + c$$

$\therefore \quad y = x\log(x+1) - x + \log(x+1) + c$ ... (i)

Since $y(0) = 3$, i.e., $y = 3$ when $x = 0$

$\therefore \quad 3 = 0 + c \Rightarrow c = 3$

$\therefore \quad y = x\log(x+1) + \log(x+1) - x + 3 \quad \ldots$[From (i)]

$\therefore \quad y = (x+1)\log(x+1) - x + 3$

Question 32

A card is drawn at random from a well shuffled pack of 52 cards. The probability that it is black card or face card is

Options:

A. $\frac{3}{13}$

B. $\frac{5}{13}$

C. $\frac{6}{13}$

D. $\frac{8}{13}$

Answer: D

Solution:

Here, $n(S) = 52$

Let event A : A black card is drawn.

event B : A face card is drawn.

$\therefore \quad \mathrm{n(A)=26, n(B)=12, n(A \cap B)=6}$

$\therefore$ Required probability

$$\begin{aligned}&= P(A \cup B)\\&= P(A)+P(B)-P(A\cap B)\\&= \frac{26}{52}+\frac{12}{52}-\frac{6}{52}\\&= \frac{32}{52}=\frac{8}{13}\end{aligned}$$

Question 33

If $\bar{a}=2\hat{i}+3\hat{j}-4\hat{k}$ and $\bar{b}=\hat{i}-\hat{j}-\hat{k}$, then the projection of $\bar{b}$ in the direction of $\bar{a}$ is

Options:

A. $\frac{1}{\sqrt{29}}$

B. $\frac{2}{\sqrt{3}}$

C. $\frac{5}{\sqrt{3}}$

D. $\frac{3}{\sqrt{29}}$

Answer: D

Solution:

Projection of $\bar{b}$ in the direction of $\bar{a}=\frac{\bar{a}\cdot\bar{b}}{|\bar{a}|}$

$$\begin{aligned}&= \frac{(2\hat{i}+3\hat{j}-4\hat{k})\cdot(\hat{i}-\hat{j}-\hat{k})}{\sqrt{2^2+3^2+(-4)^2}}\\&= \frac{2-3+4}{\sqrt{4+9+16}}=\frac{3}{\sqrt{29}}\end{aligned}$$

Question 34

If $f(x)=\begin{cases} e^{\cos x}\sin x & , \text{ for } |x|\le 2 \\ 2, & \text{otherwise}\end{cases}$**, then** $\int_{-2}^{3} f(x)dx$ **is equal to**

Options:

A. 0

B. 2

C. 1

D. 3

Answer: B

Solution:

$$\int_{-2}^{3} f(x)dx = \int_{-2}^{2} f(x)dx + \int_{2}^{3} f(x)dx$$

$$= \int_{-2}^{2} e^{\cos x}\sin x\, dx + \int_{2}^{3} 2\, dx$$

Since $e^{\cos x}\sin x$ is an odd function.

$\therefore \quad \int_{-2}^{3} f(x)dx = 0 + 2(3-2) = 2$

Question 35

The equation of the line passing through the point $(-1, 3, -2)$ **and perpendicular to each of the lines** $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ **and** $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$ **is**

Options:

A. $\frac{x+1}{2}=\frac{y-3}{7}=\frac{z+2}{4}$

B. $\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$

C. $\frac{x-1}{2}=\frac{y+3}{-7}=\frac{z+2}{4}$

D. $\frac{x-1}{2}=\frac{y+3}{7}=\frac{z-2}{4}$

Answer: B

Solution:

Let a, b, c be the direction ratios of the required line. Since the line is perpendicular to the lines with d.r.s. $1, 2, 3$ and $-3, 2, 5$.

$\therefore$ $\mathrm{a}+2\mathrm{~b}+3\mathrm{c}=0$ .... (i)

and $-3a+2b+5c=0$ .... (ii)

$\Rightarrow \frac{\mathrm{a}}{2}=\frac{\mathrm{b}}{-7}=\frac{\mathrm{c}}{4}$ ...[From (i) and (ii)]

$\therefore$ Equation of the required line is

$\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$

Question 36

Let $\mathrm{f}: \mathrm{R} \rightarrow \mathrm{R}$ be a function such that $\mathrm{f}(x)=x^3+x^2 \mathrm{f}'(1)+x \mathrm{f}''(2)+6, x \in \mathrm{R}$, then $\mathrm{f}(2)$ is

Options:

A. 30

B. -4

C. -2

D. 8

Answer: C

Solution:

$$f(x) = x^3 + x^2 f'(1) + x f''(2) + 6$$
$$\therefore \quad f'(x) = 3x^2 + 2x f'(1) + f''(2) \quad \text{... (i)}$$
$$\therefore \quad f''(x) = 6x + 2f'(1) \quad \text{... (ii)}$$

Substituting $x = 1$ in (i), we get

$$f'(1) = 3(1)^2 + 2(1)f'(1) + f''(2)$$
$$\Rightarrow f'(1) + f''(2) = -3 \quad \text{... (iii)}$$

Substituting $x = 2$ in (ii), we get

$$f''(2) = 6(2) + 2f'(1)$$
$$\Rightarrow f''(2) = 12 + 2f'(1) \quad \text{... (iv)}$$

From (iii) and (iv), we get

$$f'(1) + 12 + 2f'(1) = -3$$
$$\Rightarrow 3f'(1) = -15$$
$$\Rightarrow f'(1) = -5$$
$$\text{From (iii), } -5 + f''(2) = -3$$
$$\Rightarrow f''(2) = 2$$
$$\therefore \quad f(2) = 2^3 + 2^2(-5) + 2(2) + 6$$
$$= 8 - 20 + 4 + 6 = -2$$

Question 37

If $A(1, 4, 2)$ and $C(5, -7, 1)$ are two vertices of triangle ABC and $G\left(\frac{4}{3}, 0, \frac{-2}{3}\right)$ is centroid of the triangle ABC, then the mid point of side BC is

Options:

A. $\left(-2, -2, \frac{3}{2}\right)$

B. $\left(2, 2, \frac{3}{2}\right)$

C. $\left(\frac{3}{2}, 2, -2\right)$

D. $\left(\frac{3}{2}, -2, -2\right)$

Answer: D

Solution:

Let B $\equiv (x_1, y_1, z_1)$

Co-ordinates of centroid

$\equiv \left(\frac{1+x_1+5}{3}, \frac{4+y_1-7}{3}, \frac{2+z_1+1}{3}\right)$

$\Rightarrow \left(\frac{4}{3}, 0, \frac{-2}{3}\right) \equiv \left(\frac{6+x_1}{3}, \frac{y_1-3}{3}, \frac{3+z_1}{3}\right)$

$\Rightarrow x_1 = -2, y_1 = 3, z_1 = -5$

$\therefore \quad \mathrm{B} \equiv (-2, 3, -5)$

Midpoint side BC $= \left(\frac{-2+5}{2}, \frac{3-7}{2}, \frac{-5+1}{2}\right)$

$= \left(\frac{3}{2}, -2, -2\right)$

Question 38

The base of an equilateral triangle is represented by the equation $2x - y - 1 = 0$ and its vertex is $(1, 2)$, then the length (in units) of the side of the triangle is

Options:

A. $\sqrt{\frac{20}{3}}$

B. $\frac{2}{\sqrt{15}}$

C. $\sqrt{\frac{8}{15}}$

D. $\sqrt{\frac{15}{2}}$

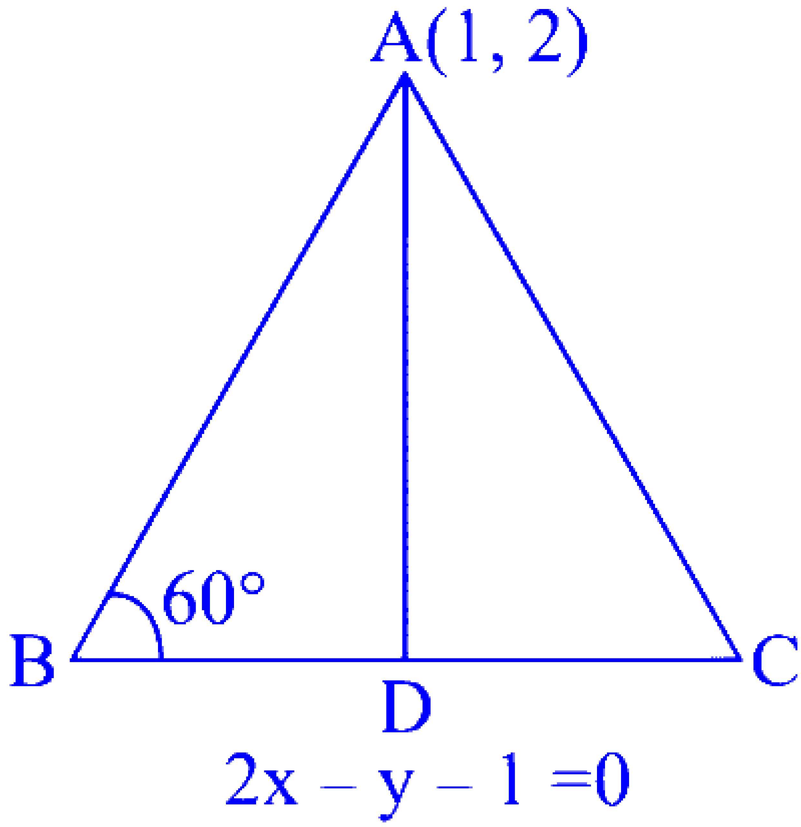
Answer: B

Solution:

$$AD = \left| \frac{2-2-1}{\sqrt{2^2+(-1)^2}} \right|$$

$$= \left| \frac{-1}{\sqrt{5}} \right|$$

$$= \frac{1}{\sqrt{5}}$$



In $\triangle ABD, \tan 60^\circ = \frac{AD}{BD}$

$$\Rightarrow \sqrt{3} = \frac{\frac{1}{\sqrt{5}}}{BD}$$

$$\Rightarrow BD = \frac{1}{\sqrt{15}}$$

$$\therefore BC = 2BD = \frac{2}{\sqrt{15}}$$

Question 39

Five persons A, B, C, D and E are seated in a circular arrangement. If each of them is given a cap of one of the three colours red, blue and green, then the number of ways of distributing the caps such that the persons seated in adjacent seats get different coloured caps, is

Options:

A. 30

B. 15

C. 60

D. 40

Answer: A

Solution:

There are 5 caps and 3 colours.

$\therefore$ At least one colour will get repeated.

As adjacent caps should be of different colours, no colour can repeat thrice.

$\therefore$ Exactly two colours will repeat twice.

$\therefore$ Colour of the caps are selected in 3 ways as follows:

Red-Red-Green-Green-Blue,

Red-Red-Green-Blue-Blue,

Red-Green-Green-Blue-Blue.

Now, while distributing the caps from above combinations, we choose any one of the 5 persons and give single colour cap. And remaining four caps can be distributed in alternate colour sequence, clock-wise or anticlock-wise.

This can be done in 5×2 ways.

$\therefore$ Required number of ways $= 3 \times 5 \times 2 = 30$.

Question 40

The distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$ is

Options:

A. 13 units.

B. 12 units.

C. 5 units.

D. 16 units.

Answer: A

Solution:

Let $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = \lambda$

$\therefore$ The co-ordinates of any point on the line are

$P \equiv (3\lambda + 2, 4\lambda - 1, 12\lambda + 2)$

This point lies on the plane

$$x - y + z = 5$$

$$\therefore \quad 3\lambda + 2 - (4\lambda - 1) + 12\lambda + 2 = 5$$

$$\Rightarrow 11\lambda = 0$$

$$\Rightarrow \lambda = 0$$

$$\therefore \quad \mathrm{P} \equiv (2, -1, 2)$$

$$\text{Let } \mathrm{Q} \equiv (-1, -5, -10)$$

$$\therefore \quad \mathrm{PQ} = \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2}$$

$$= \sqrt{9 + 16 + 144}$$

$$= 13 \text{ units}$$

Question 41

Negation of inverse of the following statement pattern $(p \wedge q) \rightarrow (p \vee \sim q)$ is

Options:

A. p

B. $\sim q$

C. $\sim p$

D. q

Answer: B

Solution:

Inverse of $(p \wedge q) \rightarrow (p \vee \sim q)$ is

$\sim (p \wedge q) \rightarrow \sim (p \vee \sim q)$
$\equiv \sim [(p \wedge q)] \vee \sim (p \vee \sim q) \ldots [p \rightarrow q \equiv \sim p \vee q]$

$\equiv (p \wedge q) \vee (\sim p \wedge q)$ ...[De Morgan's law]

$\equiv (q \wedge p) \vee (q \wedge \sim p)$ ...[Commutative law]

$\equiv q \wedge (p \vee \sim p)$ .... [Distributive law]

$\equiv q \wedge T$ ...[Complement law]

$\equiv q$ .... [Identity law]

$\therefore$ Negation of inverse of $(p \wedge q) \rightarrow (p \vee \sim q)$ is $\sim q$

Question 42

If f$(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$, then $\lim_{\alpha \to 0} \frac{f(1-\alpha)-f(1)}{\alpha^3+3\alpha} =$

Options:

A. $\frac{53}{3}$

B. $\frac{-53}{3}$

C. $\frac{52}{3}$

D. $\frac{-52}{3}$

Answer: A

Solution:

$$
\begin{aligned}
& f(x)=3x^{10}-7x^8+5x^6-21x^3+3x^2-7 \\
\therefore \quad & f'(x)=30x^9-56x^7+30x^5-63x^2+6x \\
& \Rightarrow f'(1)=30-56+30-63+6=-53
\end{aligned}
$$

Now, $\lim_{\alpha\to 0} \frac{f(1-\alpha)-f(1)}{\alpha^3+3\alpha}$

$$
\begin{aligned}
& =-\lim_{\alpha\to 0} \frac{f(1-\alpha)-f(1)}{(1-\alpha)-1} \times \frac{1}{\alpha^2+3} \\
& =-f'(1) \times \frac{1}{3}=\frac{53}{3}
\end{aligned}
$$

Question 43

The area (in sq. units) of the region bounded by curves $y=3x+1, y=4x+1$ and $x=3$ is

Options:

A. $\frac{7}{2}$

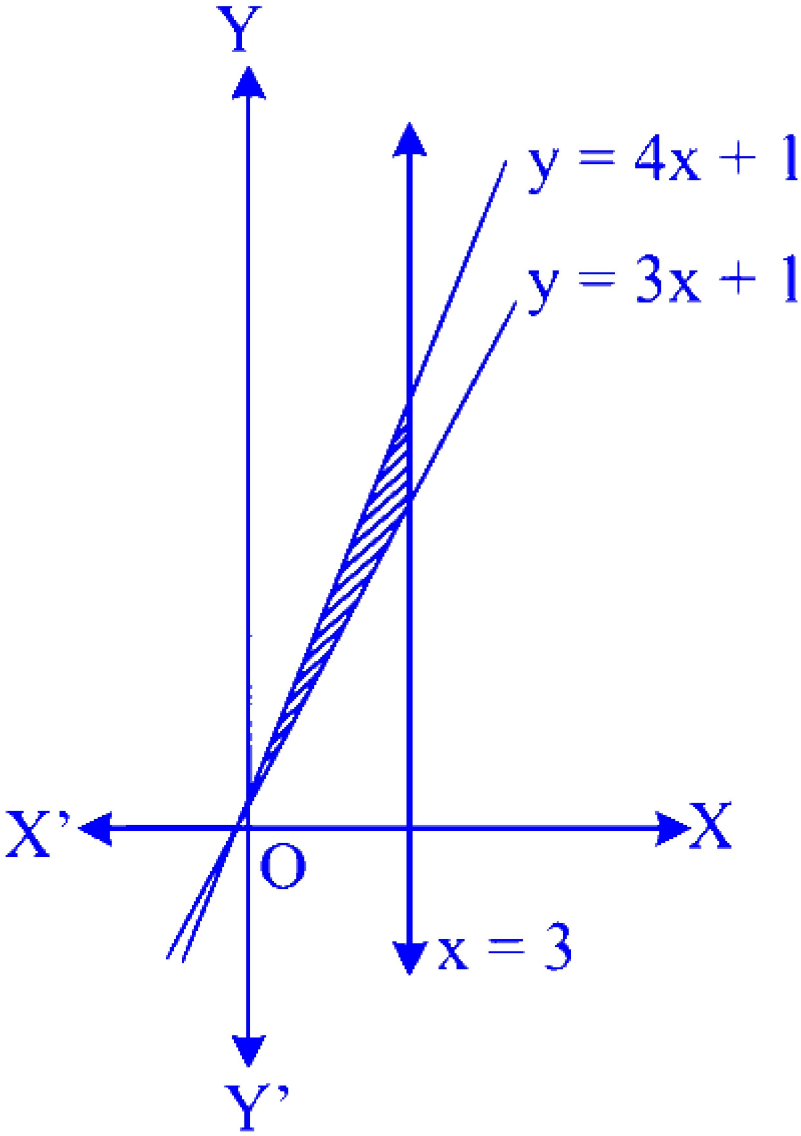
B. $\frac{9}{5}$

C. $\frac{9}{2}$

D. $\frac{7}{5}$

Answer: C

Solution:



$$\text{Required area } = \int_0^3 [4x + 1 - (3x + 1)]dx$$

$$= \int_0^3 x\, dx$$

$$= \left[\frac{x^2}{2}\right]_0^3$$

$$= \frac{9}{2} \text{ sq. units}$$

Question 44

Values of c as per Rolle's theorem for $f(x) = \sin x + \cos x + 6$ on $[0, 2\pi]$ are

Options:

A. $\frac{\pi}{3}, \frac{5\pi}{3}$

B. $\frac{\pi}{6}, \frac{5\pi}{6}$

C. $\frac{\pi}{4}, \frac{5\pi}{4}$

D. $\frac{\pi}{4}, \frac{7\pi}{4}$

Answer: C

Solution:

$\mathrm{f}(x) = \sin x + \cos x + 6$
$\therefore \quad \mathrm{f}'(x) = \cos x - \sin x$
$\text{Now, } \mathrm{f}'(\mathrm{c}) = 0$
$\Rightarrow \cos \mathrm{c} - \sin \mathrm{c} = 0$
$\Rightarrow \cos \mathrm{c} = \sin \mathrm{c}$
$\Rightarrow \tan \mathrm{c} = 1$
$\Rightarrow \mathrm{c} = \frac{\pi}{4}, \frac{5\pi}{4} \quad \ldots [\because x \in [0, 2\pi]]$

Question 45

A vector $\bar{a}$ has components 1 and $2p$ with respect to a rectangular Cartesian system. This system is rotated through a certain angle about origin in the counter clock wise sense. If, with respect to the new system, $\bar{a}$ has components 1 and $(p+1)$, then

Options:

A. $\mathrm{p} = 1$ or $\mathrm{p} = \frac{1}{3}$

B. $\mathrm{p} = -1$ or $\mathrm{p} = \frac{-1}{3}$

C. $\mathrm{p} = \frac{-1}{3}$ or $\mathrm{p} = 1$

D. $\mathrm{p} = \frac{1}{3}$ or $\mathrm{p} = -1$

Answer: C

Solution:

$$\begin{aligned}\bar{a} &= 1.\hat{i} + 2p\hat{j} \\ &= \hat{i} + 2p\hat{j}\end{aligned}$$

Let $\bar{b}$ be the vector obtained on rotation with components 1 and $(p+1)$. Then,

$$\bar{b} = \hat{i} + (p+1)\hat{j}$$
$$|\bar{a}| = |\bar{b}|$$

...[Magnitude remains unchanged after rotation]

$$\begin{aligned}&\Rightarrow |\bar{a}|^2 = |\bar{b}|^2 \\ &\Rightarrow 1 + (2p)^2 = 1 + (p+1)^2 \\ &\Rightarrow 4p^2 = p^2 + 2p + 1 \\ &\Rightarrow 3p^2 - 2p - 1 = 0 \\ &\Rightarrow (3p+1)(p-1) = 0 \\ &\Rightarrow p = -\frac{1}{3} \text{ or } p = 1\end{aligned}$$

Question 46

A line is drawn through the point $(1, 2)$ to meet the co-ordinate axes at P and Q such that it forms a $\triangle \mathrm{OPQ}$, where O is the origin. If the area of $\triangle \mathrm{OPQ}$ is least, then the slope of the line PQ is

Options:

A. -2

B. 2

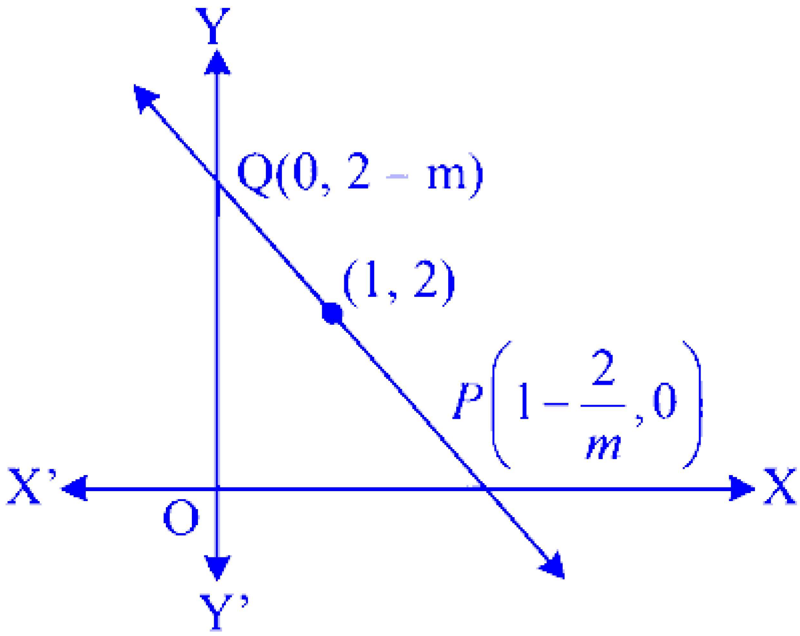
C. $\frac{-1}{2}$

D. $\frac{1}{2}$

Answer: A

Solution:

The equation of line PQ passing through $(1, 2)$ is $y - 2 = \mathrm{m}(x - 1)$



$$A(\triangle OPQ) = \frac{1}{2} \times OP \times OQ$$

$$= \frac{1}{2}\left(1 - \frac{2}{m}\right)(2 - m)$$

$$= \frac{1}{2}\left(4 - m - \frac{4}{m}\right)$$

$\therefore \quad A = 2 - \frac{m}{2} - \frac{2}{m}$

$\therefore \quad \frac{dA}{dm} = -\frac{1}{2} + \frac{2}{m^2}$

Now, $\frac{dA}{dm} = 0$

$\Rightarrow \quad -\frac{1}{2} + \frac{2}{m^2} = 0$

$\Rightarrow m^2 = 4$

$\Rightarrow m = \pm 2$

$\frac{d^2A}{\mathrm{dm}^2} = -\frac{4}{m^3}$

At$m = 2$,

$\frac{d^2A}{d^2} < 0$

At $m = -2$,

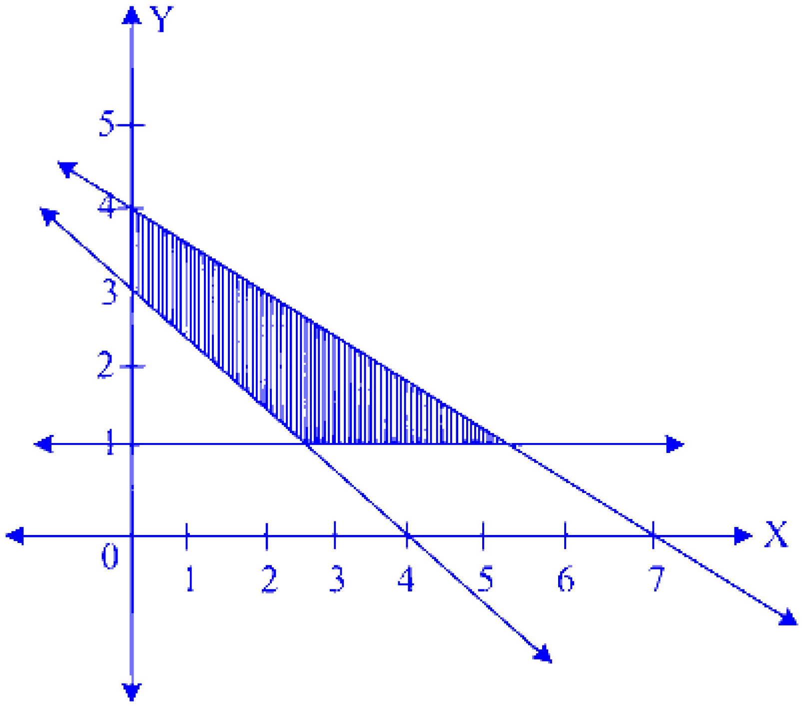
$$\frac{d^2A}{dm^2} > 0$$

$\therefore$ Area of $\triangle$OPQ will be least at $\mathrm{m} = -2$

$\Rightarrow$ Slope of the line $PQ = -2$

Question 47

If feasible region is as shown in the figure, then related inequalities are



Options:

A. $3x+4y \geq 12, 4x+7y \leq 28, y \leq 1, x \geq 0, y \geq 0$

B. $3x+4y \geq 12, 4x+7y \leq 28, y \geq 1, x \geq 0, y \geq 0$

C. $3x+4y \leq 12, 4x+7y \leq 28, y \leq 1, x \geq 0, y \geq 0$

D. $3x+4y \leq 12, 4x+7y \geq 28, y \geq 1, x \geq 0, y \geq 0$

Answer: B

Solution:

Shaded region lies on origin side of $4x+7y=28$ and above the line $y=1$, and on non-origin side of $3x+4y=12$.

$\therefore \quad 3x+4y \geq 12, 4x+7y \leq 28, y \geq 1, x \geq 0, y \geq 0$

Question 48

The general solution of the equation $3\sec^2\theta = 2\operatorname{cosec}\theta$ is

Options:

A. $n\pi + \frac{\pi}{4}, n \in Z$

B. $2n\pi + (-1)^n \frac{\pi}{12}, n \in Z$

C. $n\pi + (-1)^n \frac{\pi}{6}, n \in Z$

D. $n\pi + (-1)^n \frac{\pi}{3}, n \in Z$

Answer: C

Solution:

$$3\sec^2\theta = 2\operatorname{cosec}\theta$$
$$\Rightarrow \frac{3}{\cos^2\theta} = \frac{2}{\sin\theta}$$
$$\Rightarrow \frac{3}{1-\sin^2\theta} = \frac{2}{\sin\theta}$$
$$\Rightarrow 2\sin^2\theta + 3\sin\theta - 2 = 0$$
$$\Rightarrow (2\sin\theta - 1)(\sin\theta + 2) = 0$$
$$\Rightarrow \sin\theta = \frac{1}{2}$$

or $\sin\theta = -2$, which is not possible

$$\therefore \quad \sin\theta = \frac{1}{2} = \sin\frac{\pi}{6}$$
$$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}, n \in Z$$

Question 49

A right circular cone has height 9 cm and radius of base 5 cm. It is inverted and water is poured into it. If at any instant, the water level rises at the rate $\frac{\pi}{A}$cm/sec. where A is area of the water surface at that instant, then cone is completely filled in

Options:

A. 70 sec.

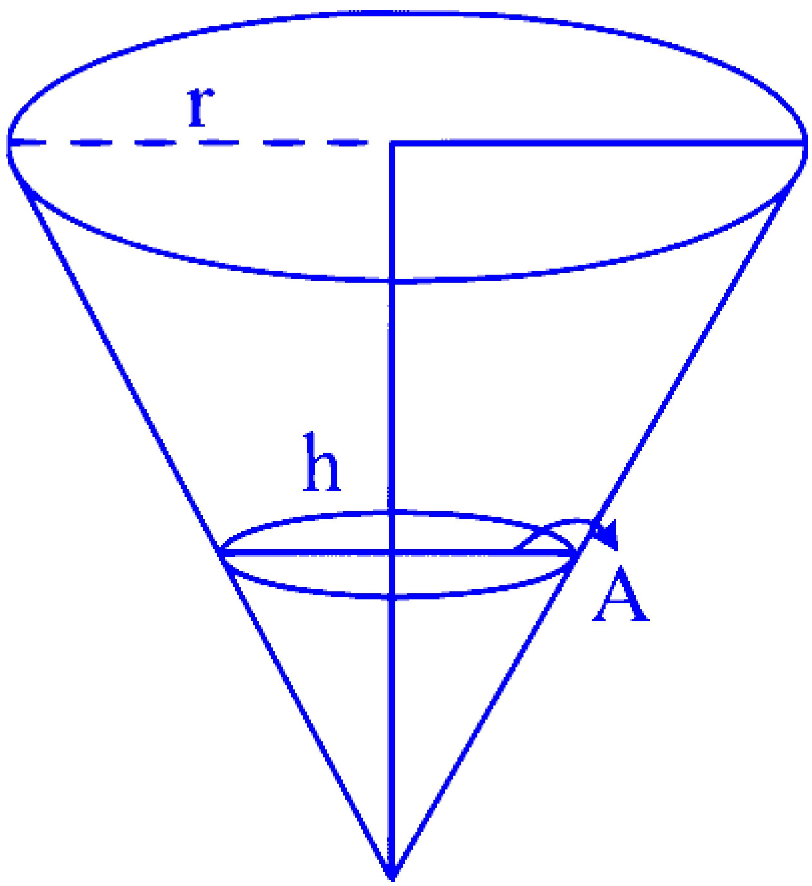
B. 75 sec.

C. 72 sec.

D. 77 sec.

Answer: B

Solution:



For the conical vessel, $h = 9$ cm, $r = 5$ cm

$\therefore$ Full volume of the vessel,

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \times 25 \times 9$$

$$= 75\pi \text{ cm}^3$$

Now, $\frac{h}{r} = \frac{9}{5}$

$$\therefore \quad r=\frac{5h}{9}$$

$$\therefore \quad A=\pi r^2=\pi\frac{25h^2}{81}$$

According to the given condition,

$$\frac{\mathrm{dh}}{\mathrm{dt}}=\frac{\pi}{\mathrm{A}}=\pi\frac{81}{\pi 25\ \mathrm{h}^2}=\frac{81}{25\ \mathrm{h}^2}$$

$$\therefore \quad \mathrm{h}^2\mathrm{dh}=\frac{81}{25}\mathrm{dt}$$

Integrating on both sides, we get

$$\frac{h^3}{3}=\frac{81}{25}t+c_1$$

$$\therefore \quad h^3=\frac{243}{25}t+c,\ \text{where } c=3c_1$$

Naturally, $\mathrm{h}=0$, when $\mathrm{t}=0$ and hence, $\mathrm{c}=0$

$$\therefore \quad \mathrm{h}^3=\frac{243}{25}\mathrm{t} \quad \text{.... (i)}$$

$$\therefore \quad \mathrm{V}=\frac{1}{3}\pi \mathrm{r}^2\ \mathrm{h}$$

$$=\frac{1}{3}\pi\frac{25\ \mathrm{h}^2}{81}\ \mathrm{h}$$

$$=\frac{25}{243}\pi \mathrm{h}^3$$

$$=\frac{25}{243}\pi\frac{243}{25}\mathrm{t} \quad \text{...[From (i)]}$$

$$\therefore \quad \mathrm{V}=\pi \mathrm{t}$$

But volume of vessel, $\mathrm{V}=75\pi$

$$\therefore \quad \pi \mathrm{t}=75\pi$$

$$\therefore \quad \mathrm{t}=75\ \text{seconds.}$$

Question 50

If θ is angle between the vectors $\bar{a}$ and $\bar{b}$ where $|\bar{a}| = 4$, $|\bar{b}| = 3$ and $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$, then $|(\bar{a} - \bar{b}) \times (\bar{a} + \bar{b})|^2 + 4(\bar{a} \cdot \bar{b})^2$ has the value

Options:

A. 576

B. 24

C. 144

D. 12

Answer: A

Solution:

$|(\bar{a} - \bar{b}) \times (\bar{a} + \bar{b})|^2 + 4(\bar{a}.\bar{b})^2$

$= \left|(\bar{a} \times \bar{a}) + \left(\bar{a} \times \bar{b}\right) - \left(\bar{b} \times \bar{a}\right) - \left(\bar{b} \times \bar{b}\right)\right|^2 + 4\left(\bar{a}.\bar{b}\right)^2$

$= |(\bar{a} \times \bar{b}) - (\bar{b} \times \bar{a})|^2 + 4(\bar{a} \cdot \bar{b})^2$

$= |2(\bar{a} \times \bar{b})|^2 + 4(\bar{a} \cdot \bar{b})^2 \ldots [(\bar{a} \times \bar{b}) = -(\bar{b} \times \bar{a})]$

$= 4\left|\left(\bar{a} \times \bar{b}\right)\right|^2 + 4\left(\bar{a}.\bar{b}\right)^2$

$= 4\left[\left|\bar{a} \times \bar{b}\right|^2 + \left(\bar{a}.\bar{b}\right)^2\right]$

$= 4|\bar{a}|^2|\bar{b}|^2$

$= 4(4)^2(3)^2$

$= 576$

Question 51

Identify cationic sphere complex from following.

Options:

A. Tetraaminecopper(II) ion

B. Tetracyanonickelate(II) ion

C. Trioxalatocobaltate(III) ion

D. Triamminetrinitrocobalt(III)

Answer: A

Solution:

Tetraaminecopper(II) ion : $[Cu(NH_3)_4]^{2+}$

Tetracyanonickelate(II) ion : $[Ni(CN)_4]^{2-}$

Trioxalatocobaltate(III) ion : $[Co(C_2O_4)_3]^{3-}$

Triamminetrinitrocobalt(III) : $[Co(NH_3)_3(NO_2)_3]$

Tetraaminecopper(II) ion is a cationic sphere complex.

Question 52

Identify substrate 'A' in the following conversion.

$$\underset{}{A} + \underset{\text{(excess)}}{CH_3I} \xrightarrow{\Delta} C_2H_5(CH_3)_3\overset{+}{N}I^-$$

Options:

A. $C_2H_5NO_2$

B. C_2H_5CN

C. $C_2H_5NH_2$

D. CH_3CONH_2

Answer: C

Solution:

$$C_2H_5NH_2 + \underset{\text{(excess)}}{CH_3I} \xrightarrow{\Delta} \underset{\text{Tetraalkylammonium iodide}}{C_2H_5(CH_3)_3NI^-}$$

The reaction is known as exhaustive alkylation of amines.

Question 53

Identify FALSE statement regarding adsorption from following.

Options:

A. It takes place due to unbalanced forces acting on the surface of solid or liquid.

B. During adsorption surface energy of adsorbent increases.

C. It is caused by van der Waals forces.

D. It is an exothermic process.

Answer: B

Solution:

During adsorption, surface energy (surface tension) of the adsorbent decreases.

Question 54

Find the rate of formation of $NO_{2(g)}$ in the following reaction.

$$2\,N_2O_{5(g)} \rightarrow 4NO_{2(g)} + O_{2(g)}$$
$$\left[\frac{-d\left[N_2O_5\right]}{dt} = 0.02 \text{ mol dm}^{-3}\right]$$

Options:

A. 0.01 mol dm^{-3}

B. 0.02 mol dm^{-3}

C. 0.03 mol dm^{-3}

D. 0.04 mol dm^{-3}

Answer: D

Solution:

$$\text{Rate of reaction} = -\frac{1}{2}\frac{d[N_2O_5]}{dt}$$
$$= +\frac{1}{4}\frac{d[NO_2]}{dt} = \frac{d[O_2]}{dt}$$
$$\text{Rate of formation of } NO_2 = \frac{d[NO_2]}{dt}$$
$$= -\frac{4}{2}\frac{d[N_2O_5]}{dt}$$
$$= -2\frac{d[N_2O_5]}{dt}$$
$$= 2 \times 0.02$$
$$= 0.04 \text{ mol dm}^{-3}$$

Question 55

Which from following properties is exhibited by group 2 elements?

Options:

A. Act as inert elements in +1 state.

B. Form MH_2 type hydrides with hydrogen on heating.

C. Elements at the top in the group catch fire when kept on water.

D. Reducing power of these elements is more than group I elements.

Answer: B

Solution:

All the metals of group 2, except beryllium, when heated with hydrogen form MH_2 type hydrides. Hence, statement (B) is correct. Elements of group 2 elements act as inert elements in +2 state. Elements at the top in the group 2 do not catch fire when kept on water. Reducing power of group 2 elements is less than group 1 elements. Hence, statements (A), (C) and (D) are incorrect.

Question 56

Calculate the rate constant of the first order reaction if 20% of the reactant decomposes in 15 minutes.

Options:

A. 1.488×10^{-2} minute $^{-1}$

B. 1.881×10^{-2} minute $^{-1}$

C. 1.984×10^{-2} minute $^{-1}$

D. 1.18×10^{-2} minute $^{-1}$

Answer: A

Solution:

20% of the reactant has decomposed.

So, if $[A]_0 = 100$, then $[A]_t = 100 - 20 = 80$

$$\begin{aligned} k &= \frac{2.303}{t} \log_{10} \frac{[A]_0}{[A]_t} \\ &= \frac{2.303}{15} \log_{10} \frac{100}{80} \\ &= \frac{2.303}{15} \log_{10} \frac{5}{4} \\ &= \frac{2.303}{15} \times (\log_{10} 5 - \log_{10} 4) \\ &= \frac{2.303}{15} \times (0.699 - 0.602) \\ &= 0.01488 = 1.488 \times 10^{-2} \text{ minute }^{-1} \end{aligned}$$

Question 57

Which from following methods of structural formula representation uses conventionally a point for front carbon and a circle around it for rear carbon?

Options:

A. Andiron formula

B. Condensed formula

C. Newman projection formula

D. Fisher projection formula

Answer: C

Question 58

Identify substrate 'A' in the following sequence of reactions.

$$\mathbf{A} \xrightarrow[\Delta]{SOCl_2} \mathbf{B} \xrightarrow[C_2H_5Cl]{Na/ether} \textbf{2-Methylbutane}$$

Options:

A. Propan-1-ol

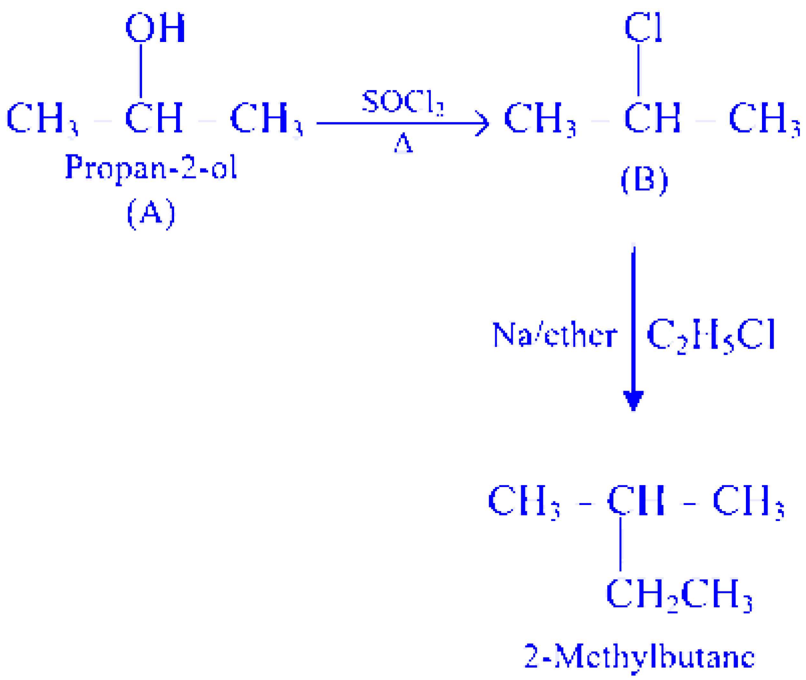
B. Propan-2-ol

C. 2-Chloropropane

D. Butan-2-ol

Answer: B

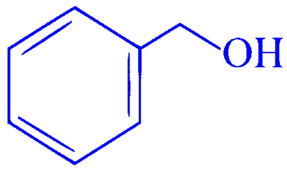
Solution:

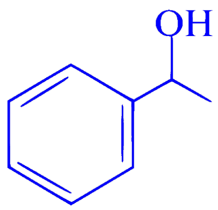


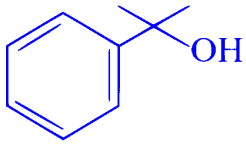
Question 59

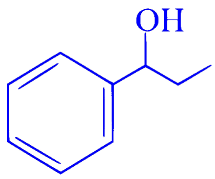
Which of the following is tertiary benzylic alcohol?

Options:

A.


B.


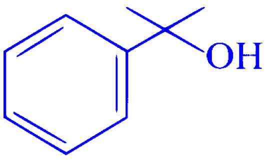
C.


D.


Answer: C

Solution:

In tertiary benzylic alcohol, $-OH$ group is bonded to a sp^3 hybridised tertiary carbon atom which is further bonded to an aromatic ring.



Question 60

Which among the following is a feature of S_N1 mechanism?

Options:

A. Single step mechanism

B. Only backside attack of nucleophile

C. Transition state contains pentacoordinate carbon

D. Formation of planar carbocation intermediate

Answer: D

Question 61

The volume of a gas is $4\ \mathrm{dm}^3$ at $0^{\circ}\mathrm{C}$. Calculate new volume at constant pressure when the temperature is increased by $10^{\circ}\mathrm{C}$.

Options:

A. $2.07\ \mathrm{dm}^3$

B. $3.21\ \mathrm{dm}^3$

C. $4.14\ \mathrm{dm}^3$

D. $6.54\ \mathrm{dm}^3$

Answer: C

Solution:

According to Charles' law,

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$ (at constant P and n)

$T_1 = 0^{\circ}\mathrm{C} = 273\ \mathrm{K}$

$T_2 = 10^{\circ}\mathrm{C} = 283\ \mathrm{K}$

$$\therefore \quad \frac{4\mathrm{dm}^3}{273\ \mathrm{K}} = \frac{V_2}{283\ \mathrm{K}}$$

$$\therefore \quad V_2 = \frac{4 \times 283}{273} = 4.1465\ \mathrm{dm}^3$$

Question 62

Which following reagent is used in Etard reaction?

Options:

A. Chromium chloride

B. Chromyl chloride

C. Chromium oxide

D. Chromic acid

Answer: B

Solution:

Methyl group in methyl benzene (or methyl arene) is oxidized by oxidizing agent chromyl chloride in carbon disulfide as solvent, to form a chromium complex, from which the corresponding benzaldehyde is obtained on acid hydrolysis. This reaction is known as Etard reaction.

Question 63

Which of the following processes exhibits increase in internal energy?

Options:

A. Adiabatic compression of gas.

B. Adiabatic expansion of gas.

C. Isothermal expansion of gas.

D. Isothermal compression of gas.

Answer: A

Solution:

In adiabatic process, there is no exchange of heat between system and its surroundings. So, $-\Delta U = -W$ or $\Delta U = W$ During compression work is done on the system by the surroundings. Therefore, W is positive. Thus, the internal energy would increase during the adiabatic compression of a gas.

Question 64

Calculate E^0_{cell} if the equilibrium constant for following reaction is 1.2×10^6.

$$2Cu^+_{(aq)} \longrightarrow Cu^{++}_{(aq)} + Cu_{(s)}$$

Options:

A. 0.36 V

B. -0.36 V

C. -0.18 V

D. 0.18 V

Answer: A

Solution:

$$E^o_{cell} = \frac{0.0592}{n}\log_{10} K \text{ at } 298 \text{ K}$$
$$Cu^+_{(aq)} + e^- \longrightarrow Cu_{(s)}$$
$$Cu^+_{(aq)} \longrightarrow Cu^{2+}_{(aq)} + e^-$$
$$2Cu^+_{(aq)} \longrightarrow Cu^{2+}_{(aq)} + Cu_{(s)}$$
$$\therefore \quad n = 1$$
$$\therefore \quad E^\circ_{cell} = \frac{0.0592}{1}\log_{10}\left(1.2 \times 10^6\right)$$
$$= 0.0592\left(\log 1.2 + \log 10^6\right)$$
$$= 0.0592(0.079 + 6)$$
$$= 0.0592 \times 6.079$$
$$= 0.36 \text{ V}$$

Question 65

Which from following alloys is used to make statues?

Options:

A. Nichrome

B. Stainless steel

C. Bronze

D. Cupra-nickel

Answer: C

Solution:

Bronze, an alloy of copper and tin, is used for making statues.

Question 66

Which of the following molecules does NOT obey octet rule?

Options:

A. CCl_4

B. Cl_2

C. O_2

D. BeF_2

Answer: D

Solution:

In BeF_2, Be has 4 electrons in its valence shell. Thus, it does not obey octet rule.

Question 67

Calculate van't Hoff factor of $\mathrm{K_2SO_4}$ if $0.1\ \mathrm{m}$ aqueous solution of $\mathrm{K_2SO_4}$ freezes at $-0.43^{\circ}\mathrm{C}$ and cryoscopic constant of water is $1.86\ \mathrm{K\ kg\ mol^{-1}}$.

Options:

A. 2.3

B. 2.7

C. 3.1

D. 3.5

Answer: A

Solution:

$$\begin{aligned} & \Delta T_f = iK_f m \\ \therefore\ & 0.43 = i \times 1.86 \times 0.1 \\ \therefore\ & i = \frac{0.43}{1.86 \times 0.1} = 2.3 \end{aligned}$$

[Note: In the question, the freezing point of aqueous solution is changed from -0.43 K to $-0.43^{\circ}\mathrm{C}$ to apply appropriate textual concepts.]

Question 68

Find the radius of fourth orbit of hydrogen atom if its radius of first orbit is R pm.

Options:

A. R pm

B. 4 R pm

C. 9 R pm

D. 16 R pm

Answer: D

Solution:

Radius of nth orbit of H-atom, $r_n = n^2 a_0$ where a_0 = radius of the first orbit Radius of the fourth orbit of H-atom $= r_4 = (4)^2$ R pm $= 16$ R pm

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Question 69

Which noble gas element from following exhibits highest number of different oxidation states?

Options:

A. Xe

B. Kr

C. Ar

D. Ne

Answer: A

Solution:

Xenon has large atomic size and lower ionisation enthalpy compared to He, Ne, Ar and Kr. Hence, xenon exhibits highest number of different oxidation states.

Question 70

Which among the following is a pair of monocarboxylic acids?

Options:

A. Malonic acid and propionic acid

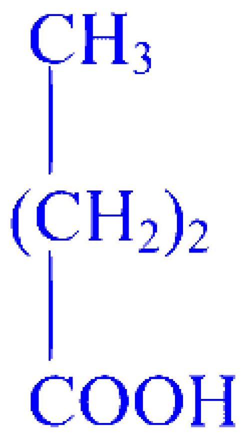
B. Valeric acid and succinic acid

C. Acetic acid and adipic acid

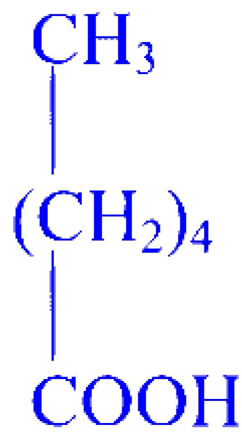
D. Butyric acid and caproic acid

Answer: D

Solution:



(n-butyric acid)



(n-caproic acid)

Propionic acid, valeric acid and acetic acid are monocarboxylic acids.

Malonic acid, succinic acid and adipic acid are dicarboxylic acids.

Question 71

Identify the overall oxidation reaction that occurs in lead storage cell during discharge.

Options:

A. $Pb^{++}_{(aq)} + SO^{--}_{4(aq)} \longrightarrow PbSO_{4(s)}$

B. $PbSO_{4(s)} + 2H_2O_{(f)} \longrightarrow PbO_{2(B)} + 4H^{+}_{(aq)} + SO^{--}_{4(aq)} + 2e^{-}$

C. $Pb_{(s)} + SO^{--}_{4(aq)} \longrightarrow PbSO_{4(s)} + 2e^{-}$

D. $PbSO_{4(s)} + 2e^- \longrightarrow Pb_{(s)} + SO^{--}_{4(aq)}$

Answer: C

Solution:

When the lead storage cell provides current (1.e., during discharge), spongy lead (Pb) is oxidised to Pb^{2+} ions and negative charge accumulates on lead plates. The Pb^{2+} ions so formed combine with SO_4^{2-} ions from H_2SO_4 to form insoluble $PbSO_4$. The overall oxidation is the sum of these two processes.

$$Pb_{(s)} \longrightarrow Pb^{2+}_{(aq)} + 2e^- \quad \text{(oxidation)}$$
$$Pb^{2+}_{(aq)} + SO^{2-}_{4\,(aq)} \longrightarrow PbSO_{4(s)} \quad \text{(precipitation)}$$

$$Pb_{(s)} + SO^{2-}_{4(ag)} \longrightarrow PbSO_{4(s)} + 2e^- \quad \text{(Overall oxidation)}$$

Question 72

What is the change in oxidation number of selenium in the following redox reaction?

$$SeO^{2-}_{3(a)} + Cl_{2(g)} + 2OH^- \longrightarrow SeO^{2-}_{4(a)} + 2Cl^-_{((a)} + H_2O$$

Options:

A. +2 to -2

B. -2 to +2

C. +4 to +6

D. +3 to +4

Answer: C

Solution:

SeO_3^{2-}	SeO_4^{2-}
$x + (-2 \times 3) = -2$ $x = +4$	$x + (-2 \times 4) = -2$ $x = +6$

Question 73

A buffer solution is prepared by mixing equimolar acetic acid and sodium acetate. If 'K_d' of acetic acid is 1.78×10^{-5}, find the pH of buffer solution.

Options:

A. 4.75

B. 8.9

C. 9.4

D. 2.6

Answer: A

Solution:

For acidic buffer,

$$\mathrm{pH} = \mathrm{pK_a} + \log_{10} \frac{[\,\mathrm{Salt}\,]}{[\,\mathrm{Acid}\,]} = \mathrm{pK_a} + \log_{10} \frac{1}{1} = \mathrm{pK_a}$$

$$\begin{aligned} \mathrm{pK_a} &= -\log_{10}\ \mathrm{K_a} = -\log_{10}\left(1.78 \times 10^{-5}\right) \\ &= -\left(\log_{10} 1.78 + \log_{10} 10^{-5}\right) \\ &= -(0.25 - 5) = 4.75 \end{aligned}$$

Therefore, pH of buffer solution $= 4.75$

Question 74

What is the number of moles of nascent hydrogen required to prepare 1 mole of methane from iodomethane?

Options:

A. ,4

B. 3

C. 2

D. 1

Answer: C

Solution:

$$\underset{\text{Methyl iodide}}{CH_3 - I} + \underset{\text{Nascent}}{2[H]} \xrightarrow{\text{Zn, HCl}} \underset{\text{Methane}}{CH_4} + HI$$

Hence, the number of moles of nascent hydrogen required to prepare 1 mole of methane from iodomethane is 2 moles.

Question 75

Which from following metal has hcp crystal structure?

Options:

A. Cu

B. Zn

C. Ag

D. Po

Answer: B

Solution:

Zn has hep crystal structure; Cu and Ag have ccp crystal structure while Po has simple cubic closed packed structure.

Question 76

One mole of an ideal gas performs 900 J of work on surrounding. If internal energy increases by 625 J, find the value of ΔH.

Options:

A. -275 J

B. 200 J

C. -150 J

D. 525 J

Answer: A

Solution:

$W = -900 \text{ J}, \Delta U = +625 \text{ J}$

Assuming constant pressure,

$$\Delta H = \Delta U + P_{ext} \Delta V$$
$$\therefore \quad \Delta H = \Delta U - W$$
$$\therefore \quad \Delta H = +625 - (-900) = 1525 \text{ J}$$

[Note: For the given question, none of the provided options is the correct answer.]

Question 77

Which of the following temperature values in Fahrenheit $({}^{\circ}\text{F})$ is equal to 50°C ?

Options:

A. 90°F

B. 100°F

C. 110°F

D. 122°F

Answer: D

Solution:

$$\begin{aligned} {}^{\circ}\mathrm{F} &= \frac{9}{5}({}^{\circ}\mathrm{C}) + 32 \\ &= \frac{9}{5}(50) + 32 \\ &= 90 + 32 = 122^{\circ}\mathrm{F} \end{aligned}$$

Question 78

Which from following formulae is of trioxalatocobaltate(III) ion?

Options:

A. $[CO_3\,(C_2O_4)]^{3-}$

B. $[Co(C_2O_4)_3]^{3+}$

C. $[Co_3\,(C_2O_4)]^{3+}$

D. $[Co(C_2O_4)_3]^{3-}$

Answer: D

Solution:

Trioxalatocobaltate(III) ion : $[Co(C_2O_4)_3]^{3-}$

Question 79

Which of the following pair of nuclides is an example of isotones?

Options:

A. ${}^{35}_{17}Cl$, ${}^{37}_{17}Cl$

B. ${}^{40}_{18}Ar$, ${}^{40}_{19}K$

C. ${}^{13}_{6}C$, ${}^{11}_{5}B$

D. ${}^{12}_{6}C$, ${}^{11}_{5}B$

Answer: D

Solution:

The atoms of different elements having same number of neutrons in their nuclei are called isotones.

${}^{12}_{6}C$ and ${}^{11}_{5}B$ are isotones.

Question 80

Identify FALSE statement from following.

Options:

A. Boiling point of sulfur is lower than oxygen.

B. Ionization enthalpy of group 16 elements gradually decreases from top to bottom.

C. Group 16 elements have lower ionization enthalpy than group 15 elements in corresponding periods.

D. Oxygen has highest electronegativity next to fluorine amongst all the elements.

Answer: A

Solution:

Melting and boiling points of elements of group 16 increase with increasing atomic number. Therefore, oxygen has low melting and boiling points as compared to sulfur.

Question 81

What is IUPAC name of crotonyl alcohol?

Options:

A. But-3-en-1-ol

B. But-2-en-1-ol

C. But-1-en-2-ol

D. But-1-en-3-ol

Answer: B

Solution:

$H_3CCH = CHCH_2OH$

Common name: Crotonyl alcohol

IUPAC name: But-2-en-1-ol

Question 82

A solution of 5.6 g non-volatile solute in 50 g solvent has elevation in boiling point 1.75 K. What is the molar mass of solute $\left(K_b = 3 \text{ K kg mol}^{-1}\right)$?

Options:

A. 192 g mol^{-1}

B. 200 g mol^{-1}

C. 184 g mol^{-1}

D. 176 g mol^{-1}

Answer: A

Solution:

$$M_2 = \frac{1000\ K_b W_2}{\Delta T_b W_1}$$
$$= \frac{1000 \times 3 \times 5.6}{1.75 \times 50} = 192\ g\ mol^{-1}$$

Question 83

What is the common name of benzene-1,3-diol?

Options:

A. Catechol

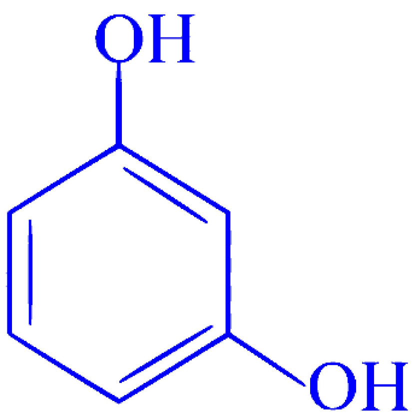
B. Resorcinol

C. Quinol

D. Pyrogallol

Answer: B

Solution:



Resorcinol
(Benzene-1,3-diol)

Question 84

Identify the CORRECT decreasing order of basic strength of compounds from following.

Options:

A. $CH_3NH_2 > (CH_3)_2NH > C_6H_5NH_2 > NH_3$

B. $(CH_3)_2NH > CH_3NH_2 > NH_3 > C_6H_5NH_2$

C. $NH_3 > CH_3NH_2 > (CH_3)_2NH > C_6H_5NH_2$

D. $C_6H_5NH_2 > (CH_3)_2NH > CH_3NH_2 > NH_3$

Answer: B

Solution:

Arylamines in general are weaker bases than ammonia and aliphatic amines.

Aliphatic amines are stronger bases than ammonia. Among aliphatic amines, 2° amine is a stronger base than 1° amine.

Hence, the correct decreasing order of basic strength is:

$(CH_3)_2NH > CH_3NH_2 > NH_3 > C_6H_5NH_2$

Question 85

Calculate the work done in the following reaction at 300 K and at constant pressure.

$$\left(R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}\right)$$

$$4HCl_{(g)} + O_{2(g)} \rightarrow 2Cl_{2(g)} + 2H_2O_{(g)}$$

Options:

A. -7482 J

B. -4988 J

C. 2494 J

D. 3200 J

Answer: C

Solution:

$$\begin{aligned} W &= -\Delta n_g RT \\ &= -(4-5) \times 8.314 \times 300 \\ &= 1 \times 8.314 \times 300 \\ &= 2494.2 \text{ J} \end{aligned}$$

Question 86

The solubility product of $Mg(OH)_2$ is 1.8×10^{-11} at 298 K. What is its solubility in $mol\ dm^{-3}$?

Options:

A. 1.650×10^{-4}

B. 2.120×10^{-4}

C. 3.184×10^{-4}

D. 4.550×10^{-4}

Answer: A

Solution:

$$Mg(OH)_{2(s)} \rightleftharpoons Mg^{2+}_{(aq)} + 2OH^{-}_{(aq)}$$

Here, $x = 1, y = 2$

$$\therefore \quad K_{sp} = x^x y^y S^{x+y} = (1)^1(2)^2 S^{1+2} = 4S^3$$

$$\therefore \quad S = \sqrt[3]{\frac{K_{sp}}{4}} = \sqrt[3]{\frac{1.8 \times 10^{-11}}{4}}$$

$$= \sqrt[3]{4.5 \times 10^{-12}} = 1.650 \times 10^{-4}$$

We know that, $\sqrt[3]{1} = 1$ and $\sqrt[3]{8} = 2$

$\sqrt[3]{1} < \sqrt[3]{4.5} < \sqrt[3]{8}$

Therefore, $\sqrt[3]{8} < 2$.

Only option (A) satisfies this condition.

Question 87

If K_{sp} is solubility product of $Al(OH)_3$, its solubility is expressed by formula,

Options:

A. $\sqrt[3]{\frac{4}{K_{sp}}}$

B. $\sqrt[3]{\frac{K_{sp}}{4}}$

C. $\sqrt[4]{\frac{K_{sp}}{27}}$

D. $\sqrt[4]{K_{sp} \times 27}$

Answer: C

Solution:

$$Al(OH)_{3(s)} \rightleftharpoons Al^{3+}_{(aq)} + 3OH^{-}_{(aq)}$$

Here, $x = 1, y = 3$

$\therefore \quad K_{sp} = x^x y^y S^{x+y} = (1)^1 (3)^3 S^{1+3} = 27 S^4$

$\therefore \quad S = \sqrt[4]{\frac{K_{sp}}{27}}$

Question 88

Which from following is the slope of the graph of $[A]_t$ versus time for zero order reaction?

Options:

A. $-k$

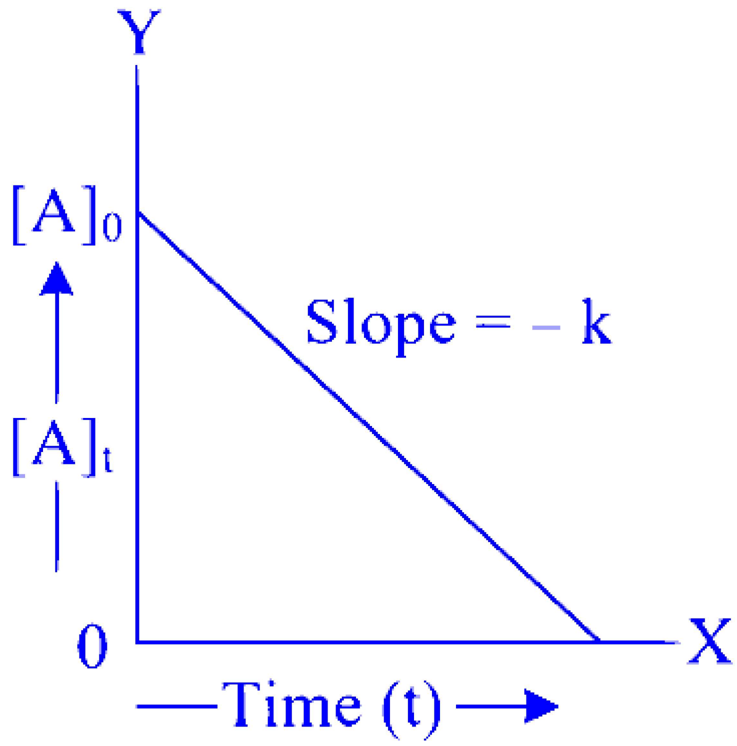
B. k

C. $\frac{k}{2.303}$

D. $\frac{-k}{2.303}$

Answer: A

Solution:



Variation of [A]t with time (t) for a zero order reaction

Question 89

Which from following statements is NOT true about natural rubber?

Options:

A. It is a linear polymer.

B. In this polymer chain has coiled structure.

C. In this polymer chains are held together by weak van der Waals forces.

D. It is obtained from chloroprene.

Answer: D

Solution:

Natural rubber is a high molecular mass linear polymer of isoprene. It consists of various chains held together by weak van der Waals forces and has coiled structure.

Question 90

What is the molal elevation constant if one gram mole of a nonvolatile solute is dissolved in $1 \mathrm{~kg}$ of ethyl acetate? $\left(\Delta \mathrm{T}_{\mathrm{b}}=x \mathrm{~K}\right)$

Options:

A. $x \mathrm{~K} \mathrm{~kg} \mathrm{~mol}^{-1}$

B. $\frac{x}{2} \mathrm{~K} \mathrm{~kg} \mathrm{~mol}^{-1}$

C. $2 x \mathrm{~K} \mathrm{~kg} \mathrm{~mol}^{-1}$

D. $3 x \mathrm{~K} \mathrm{~kg} \mathrm{~mol}^{-1}$

Answer: A

Solution:

$$\mathrm{M}_2=\frac{1000 \mathrm{~K}_{\mathrm{b}} \mathrm{W}_2}{\Delta \mathrm{T}_{\mathrm{b}} \mathrm{W}_1}$$

$$\therefore \quad \mathrm{K}_{\mathrm{b}}=\frac{\mathrm{M}_2 \Delta \mathrm{T}_{\mathrm{b}} \mathrm{W}_1}{1000 \mathrm{~W}_2}$$

Now, $\mathrm{W}_2=$ one gram mole $=\mathrm{M}_2 \mathrm{~g}$

$\mathrm{W}_1=1 \mathrm{~kg}=1000 \mathrm{~g}$

$$\therefore \quad \mathrm{K}_{\mathrm{b}}=\frac{\mathrm{M}_2 \times x \times 1000}{1000 \times \mathrm{M}_2}=x \mathrm{~K} \mathrm{~kg} \mathrm{~mol}^{-1}$$

One gram mole solute dissolved in 1 kg solvent = 1 molal solution

When concentration of solution is 1 molal, elevation in boiling point (ΔT_b) is equal to molal elevation constant $(\mathrm{K}_{\mathrm{b}})$.

Therefore, $\mathrm{K}_{\mathrm{b}}=x \mathrm{~K} \mathrm{~kg} \mathrm{~mol}^{-1}$

Question 91

Which from following elements exhibits ferromagnetic properties?

Options:

A. Mn

B. Co

C. Zn

D. Sc

Answer: B

Solution:

Among transition metals, Fe, Co and Ni are ferromagnetic.

Question 92

Which of the following is NOT a globular protein?

Options:

A. Insulin

B. Egg albumin

C. Serum albumin

D. Keratin

Answer: D

Solution:

Insulin, egg albumin and serum albumin are globular proteins while keratin is a fibrous protein.

Question 93

Calculate the time required in second to deposit 6.35 g copper from its salt solution by passing 5 ampere current. [Molar mass of $\mathrm{Cu} = 63.5 \mathrm{~g~mol}^{-1}$]

Options:

A. 3600

B. 3700

C. 3860

D. 4000

Answer: C

Solution:

$$\mathrm{Cu}_{(s)}^{2+} + 2\mathrm{e}^{-} \longrightarrow \mathrm{Cu}_{(s)}$$

$$\text{Mole ratio } = \frac{1 \text{ mol}}{2 \text{ mole}^{-}}$$

$$\mathrm{W} = \frac{\mathrm{I(A)} \times \mathrm{t(s)}}{96500\,(\mathrm{C/mole}^{-})} \times \text{mole ratio } \times \text{ molar mass}$$

$$6.35 \mathrm{~g} = \frac{5 \times \mathrm{t}}{96500\,(\mathrm{C/mole}^{-})} \times \frac{1 \text{ mol}}{2 \text{ mole}^{-}} \times 63.5 \mathrm{~g~mol}^{-1}$$

$$\mathrm{t} = \frac{6.35 \times 96500 \times 2}{5 \times 63.5} = 3860 \text{ seconds}$$

Question 94

Which from following nanomaterial has one dimension less than 100 nm ?

Options:

A. Fibres

B. Nanoparticles

C. Thin films

D. Microcapsules

Answer: C

Solution:

'One dimension less than 100 nm' implies that the nanomaterial is a two-dimensional nanostructure. Among the given options, thin films are two-dimensional nanostructures.

Question 95

Which among the following compounds has highest boiling point?

Options:

A. Propanone

B. Ethanoic acid

C. Propan-1-ol

D. Propanal

Answer: B

Solution:

Carboxylic acids have higher boiling points than those of alkanes, ethers, alcohols aldehydes and ketones of comparable mass.

Question 96

Calculate the radius of metal atom in bcc unit cell having edge length 287 pm.

Options:

A. 124.27 pm

B. 143.51 pm

C. 101.45 pm

D. 57.4 pm

Answer: A

Solution:

For bcc unit cell, $4r = \sqrt{3}a$

$\therefore \quad r = \frac{\sqrt{3}}{4}a = \frac{1.732 \times 287}{4} = 124.27$ pm

---

Question 97

Identify A in the following reaction.

$$C_6H_{12}O_6 \xrightarrow[\text{dil. } HNO_3]{(O)} A$$

Options:

A. CHO
|
$(CHOH)_4$
|
CH_2OH

B. COOH
|
$(CHOH)_4$
|
COOH

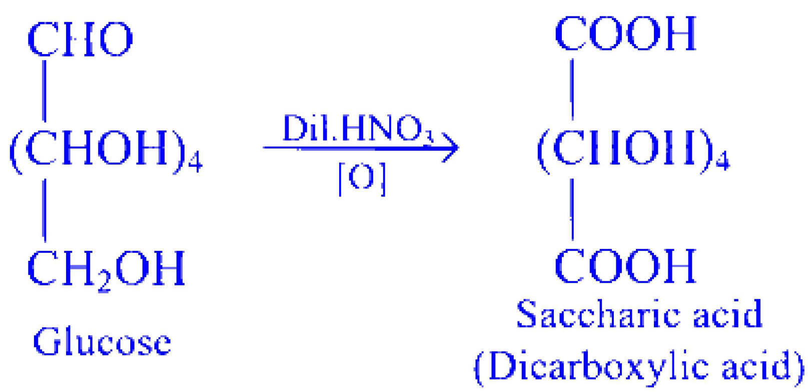
C. COOH
|
$(CHOH)_4$
|
CH_2OH

D. CH=N–OH
|
$(CHOH)_4$
|
CH_2OH

Answer: B

Solution:

Glucose on oxidation with dilute nitric acid gives saccharic acid.



Question 98

Which of the following is formed when propene is heated with chlorine at high temperature?

Options:

A. 1,2-Dichloropropane

B. 1-Chloropropane

C. 2-Chloropropane

D. 3-Chloropropene

Answer: D

Solution:

When an alkene is heated with Cl_2 at high temperature, hydrogen atom of allylic carbon is substituted with halogen atom giving allyl halide.

$$CH_2 = CH - CH_3 + Cl_2 \longrightarrow \underset{3-\text{Chloropropene}}{CH_2 = CH - CH_2Cl} + HCl$$

Alkenes form addition product, vicinal dihalide, with chlorine or bromine usually in inert solvent like CCl_4 at room temperature.

Question 99

Identify the monomer used to prepare Teflon.

Options:

A. C_2H_4

B. C_2H_3N

C. $CONH_2$ and CH_2O

D. C_2F_4

Answer: D

Solution:

The monomer used in the preparation of teflon is tetrafluoroethylene, $(CF_2 = CF_2)$.

Question 100

Calculate the number of atoms present in unit cell if an element having molar mass $23 \mathrm{~g} \mathrm{~mol}^{-1}$ and density $0.96 \mathrm{~g} \mathrm{~cm}^{-3}$.

$[a^3 \cdot N_A = 48 \mathrm{~cm}^3 \mathrm{~mol}^{-1}]$

Options:

A. 1

B. 2

C. 4

D. 6

Answer: B

Solution:

$$\text{Density } (\rho) = \frac{nM}{a^3 N_A}$$
$$0.96 = \frac{n \times 23}{48}$$
$$n = \frac{0.96 \times 48}{23} = 2.003$$

Number of atoms present in unit cell $= 2$

Physics

Question 101

Select the 'WRONG' statement out of the following.

Options:

A. Electromagnetic waves do not require any medium for their propagation.

B. Electromagnetic waves can travel through vacuum waves speed of light

C. Material medium is necessary for propagation of electromagnetic waves.

D. Electromagnetic waves are transverse in nature.

Answer: C

Solution:

Electromagnetic waves can propagate through vacuum. This can be clearly understood from Maxwell's equations for EM waves. Self-sustaining electric and magnetic fields enable waves to travel through empty space.

Question 102

Two cells E_1 and E_2 having equal EMF 'E' and internal resistances r_1 and r_2 $(r_1 > r_2)$ respectively are connected in series. This combination is connected to an external resistance 'R'. It is observed that the potential difference across the cell E_1 becomes zero. The value of 'R' will be

Options:

A. $r_1 - r_2$

B. $r_1 + r_2$

C. $\frac{r_1 - r_2}{2}$

D. $\frac{r_1 + r_2}{2}$

Answer: A

Solution:

The total current in the circuit is

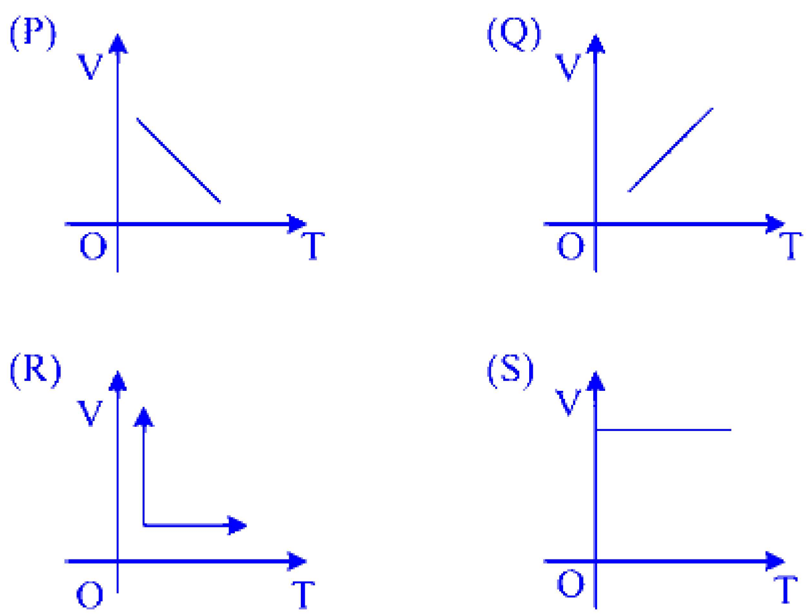
$I = \frac{2E}{r_1 + r_2 + R}$ ... (1) .... (Given cells are in series, $E + E = 2E$)

Now the potential drop across the first cell is $V_1 = E - r_1 = 0$

$$\therefore \quad E - \left(\frac{2E}{r_1 + r_2 + R}\right) \times r_1 = 0$$
$$\frac{2E}{r_1 + r_2 + R} = \frac{E}{r_1} \Rightarrow 2r_1 = r_1 + r_2 + R$$
$$\therefore \quad R = r_1 - r_2$$

Question 103

Which one of the following represents correctly the variation of volume (V) of an ideal gas with temperature (T) under constant pressure conditions?



Options:

A. P

B. Q

C. R

D. S

Answer: B

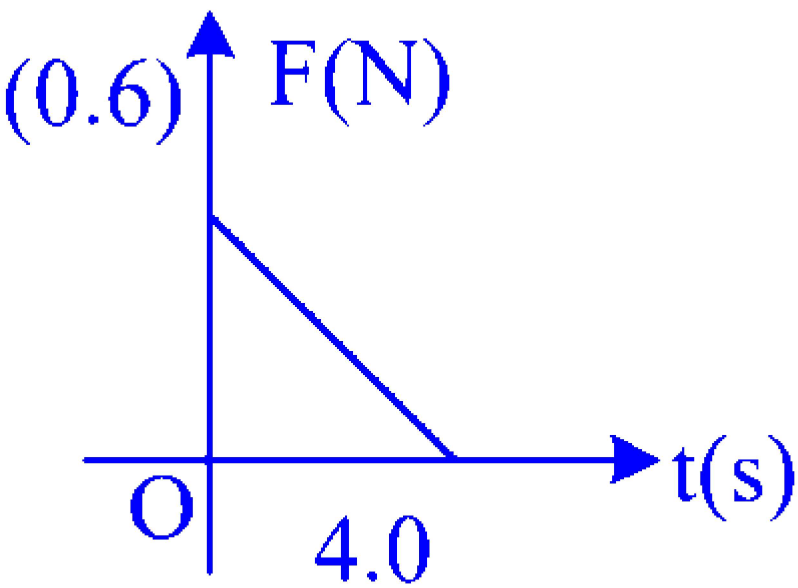
Solution:

According to Charles's Law, the volume of an ideal gas is directly proportional to its absolute temperature when pressure is held constant. i.e., $V \propto T$

This is depicted by graph Q.

Question 104

Using variation of force and time given below, final velocity of a particle of mass 2 kg moving with initial velocity 6 m/s will be



Options:

A. 10 m/s

B. 5 m/s

C. 12 m/s

D. 0 m/s

Answer: C

Solution:

Now, using the impulse-momentum theorem to find the final velocity

$12 = 2 \times (-6)$

$v - 6 = \frac{12}{2} = 6$

$v = 12 \mathrm{~m/s}$

So, with the given force-time data $(0, 6)$ and $(4, 0)$, the final velocity of the particle is $12 \mathrm{~m/s}$

Question 105

dQ is the heat energy supplied to an ideal gas under isochoric conditions. If dU and dW denote the change in internal energy and the work done respectively then

Options:

A. $\mathrm{dQ} = \mathrm{dW}$

B. $\mathrm{dQ} > \mathrm{dU}$

C. $\mathrm{dQ} < \mathrm{dU}$

D. $\mathrm{dQ} = \mathrm{dU}$

Answer: D

Solution:

Under isochoric (constant volume) conditions, the heat energy (dQ) supplied to an ideal gas contributes solely to the change in its internal energy (dU).

Mathematically, this can be expressed as dQ = dU + dW

$\Rightarrow \mathrm{dQ} = \mathrm{dU} \quad (\because \mathrm{dW} = 0)$

Question 106

A black body at temperature $127^{\circ}\mathrm{C}$ radiates heat at the rate of $5\ \mathrm{cal/cm^2\ s}$. At a temperature $927^{\circ}\mathrm{C}$, its rate of emission in units of $\mathrm{cal/cm^2\ s}$ will be

Options:

A. 405

B. 35

C. 45

D. 350

Answer: A

Solution:

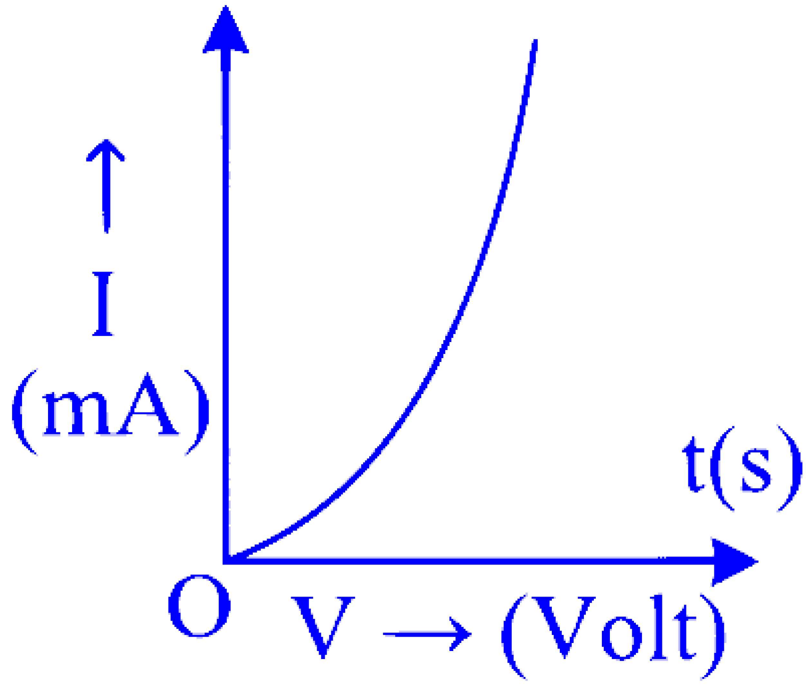
From Stefan – Boltzmann's Law

$$E = \sigma T^4$$
$$\Rightarrow E \propto T^4$$

$$\therefore \quad \frac{\mathrm{E_1}}{\mathrm{E_2}} = \left(\frac{\mathrm{T_1}}{\mathrm{T_2}}\right)^4 = \left(\frac{400}{1200}\right)^4$$
$$\frac{\mathrm{E_1}}{\mathrm{E_2}} = \frac{1}{81}$$
$$\mathrm{E_2} = 81 \times \mathrm{E_1}$$
$$= 81 \times 5$$
$$= 405\ \mathrm{cal/cm^2\ s}$$

Question 107

The following graph represents



Options:

A. forward bias characteristics of a solar cell

B. reverse bias characteristics of a Zener diode

C. reverse bias characteristics of a photodiode

D. forward bias characteristics of a LED

Answer: D

Solution:

With increasing voltage, the LED's current rises, leading to light emission due to charge carrier recombination.

Question 108

A rigid body rotates with an angular momentum L. If its rotational kinetic energy is made four times, its angular momentum will become

Options:

A. 4 L

B. 16 L

C. $\sqrt{2}$ L

D. 2 L

Answer: D

Solution:

Angular momentum,

$$\begin{aligned} & \mathrm{L}=\sqrt{2 \mathrm{KI}} \\ & \mathrm{K}=4 \mathrm{~K} \quad \text{(given)} \\ \therefore \quad & \mathrm{L}=\sqrt{2 \times 4 \mathrm{KI}}=2 \sqrt{2 \mathrm{KI}}=2 \mathrm{~L} \end{aligned}$$

Question 109

A radioactive sample has half-life of 5 years. The percentage of fraction decayed in 10 years will be

Options:

A. 25%

B. 50%

C. 75%

D. 100%

Answer: C

Solution:

Given:

Total time, $\mathrm{T}=10$ years

Half life, $\mathrm{T}_{1/2}=5$ years

$\therefore$ No. of half lives, $n = \frac{10}{5} = 2$

From $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

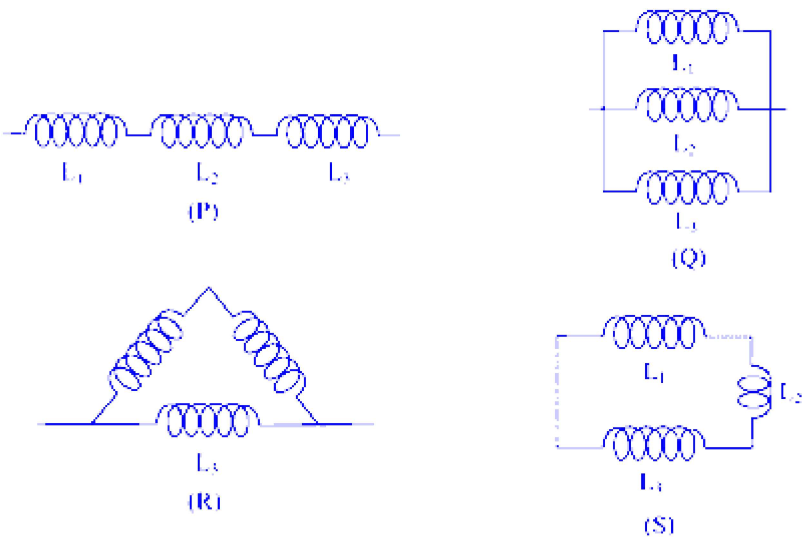
$\therefore$ $\frac{N}{N_0} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\therefore$ After 10 years, $\frac{1}{4}^{\text{th}}$ of the original substance will remain.

$\Rightarrow \left(1 - \frac{1}{4}\right) = \frac{3}{4} \times 100 = 75\%$ of the fraction would get decayed.

Question 110

Three coils of inductance $L_1 = 2H, L_2 = 3H$ and $L_3 = 6H$ are connected such that they are separated from each other. To obtain the effective inductance of 1 henry, out of the following combinations as shown in figure, the correct one is



Options:

A. S

B. P

C. R

D. Q

Answer: D

Solution:

The combinations P and S are series combinations. Hence, the effective inductance cannot be 1H.

This leaves combinations R and Q.

For combination R,

$L_1 + L_2 = 5H$

$\frac{1}{L_1+L_2} + \frac{L}{L_3} = \frac{1}{5} + \frac{1}{6} = \frac{6+5}{30} = \frac{11}{30}$

$\therefore \quad L_{eff} = \frac{30}{11} = 2.72H$

$\therefore$ The correct combination would be Q.

$$\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3}{6} + \frac{2}{6} + \frac{1}{6}$$

$\therefore \quad L_{eff} = 1H$

Question 111

A Carnot engine has the same efficiency between (i) 100 K and 600 K and (ii) TK and 960 K. The temperature T in kelvin of the sink is

Options:

A. 120

B. 160

C. 240

D. 320

Answer: B

Solution:

Efficiency of a carnot engine is $\eta = 1 - \frac{T_C}{T_H}$

For case (i),

$T_C = 100$ K and $T_H = 600$ K

$\therefore \quad \eta_1 = 1 - \frac{100}{600} = \frac{5}{6}$

For case (ii),

$T_C = TK$ and $T_H = 960$ K

$\therefore \quad \eta_2 = 1 - \frac{T}{960}$

Given $\eta_1 = \eta_2$

$$\therefore \quad \frac{5}{6} = 1 - \frac{T}{960}$$
$$\frac{5}{6} = \frac{960 - T}{960}$$

Solving for T, we get $T = 160$ K

Question 112

For which of the following substances, the magnetic susceptibility is independent of temperature?

Options:

A. Diamagnetic only.

B. Paramagnetic only.

C. Ferromagnetic only.

D. Diamagnetic and paramagnetic both.

Answer: A

Solution:

Diamagnetic substances have a small negative value for magnetic susceptibility and is independent of temperature.

Question 113

When a light of wavelength $300 \mathrm{~nm}$ fall on a photoelectric emitter, photo electrons are emitted. For another emitter light of wavelength $600 \mathrm{~nm}$ is just sufficient for liberating photoelectrons. The ratio of the work function of the two emitters is

Options:

A. 1 : 2

B. 2 : 1

C. 4 : 1

D. 1 : 4

Answer: B

Solution:

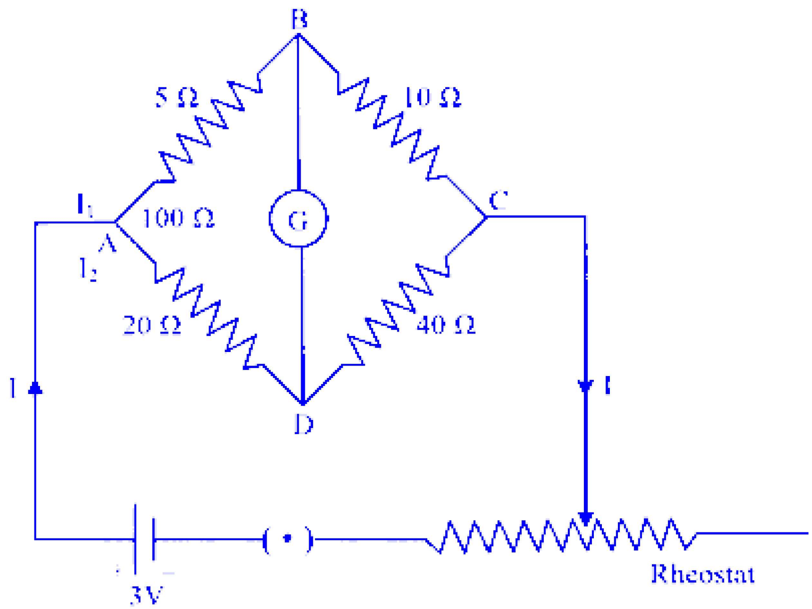
Work function $\phi_0=\frac{\mathrm{hc}}{\lambda_0}$

$$\therefore \quad \phi_0 \propto \frac{1}{\lambda_0}$$

$$\frac{\phi_{01}}{\phi_{0_2}}=\frac{\lambda_{0_2}}{\lambda_{0_1}}=\frac{600}{300}=\frac{2}{1}$$

Question 114

In a given meter bridge, the current flowing through 40Ω resistor is



Options:

A. $I_2 + I_g$

B. I_g

C. $I_2 - I_g$

D. I_2

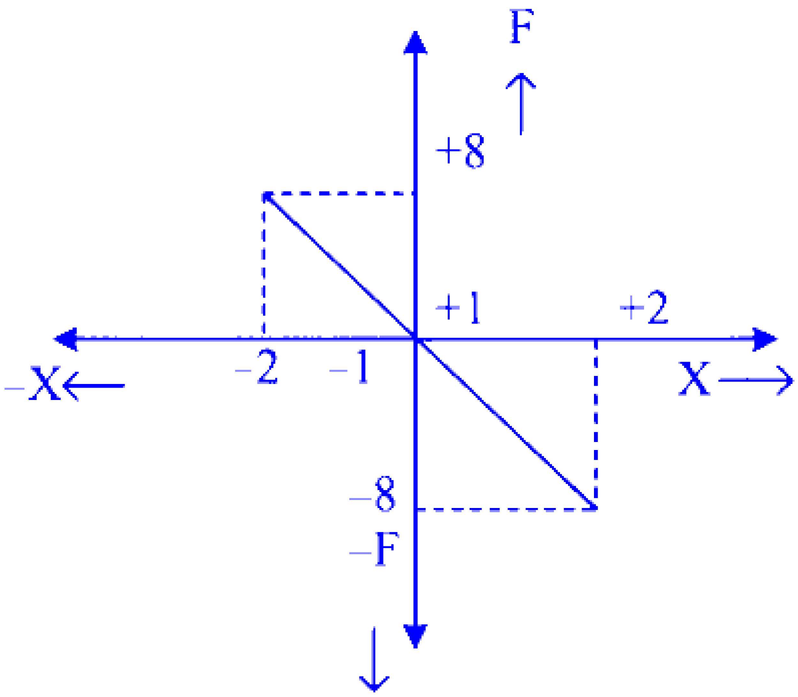
Answer: D

Solution:

As the bridge is balanced, no current will flow through the galvanometer. Hence, the current through the 40Ω resistor will be I_2.

Question 115

A body of mass 0.04 kg executes simple harmonic motion (SHM) about $x = 0$ under the influence of force F as shown in graph. The period of



Options:

A. $2\pi s$

B. $0.2\pi s$

C. πd

D. $\frac{\pi}{2}$ s

Answer: B

Solution:

$k = 400 \text{ N/m}$

$m = 0.04$

From the graph,

$K = \frac{F}{x} = \frac{8}{2} = 4$

From $T = 2\pi\sqrt{\frac{M}{K}}$,

we get

$T = 2\pi\sqrt{\frac{0.04}{4}} = 0.2\pi s$

Question 116

A large number of water droplets each of radius 't' combine to form a large drop of Radius 'R'. If the surface tension of water is 'T' & mechanical equivalent of heat is 'J' then the rise in temperature due to this is

Options:

A. $\frac{2\,T}{rJ}$

B. $\frac{3\,T}{RJ}$

C. $\frac{3T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$

D. $\frac{2\,T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$

Answer: C

Solution:

Radius of each droplet = r

Radius of the drop = R

As volume remains constant,

$$n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$
$$\therefore \quad n = \frac{R^3}{r^3}$$

Decrease in surface area $= 4\pi r^2 n - 4\pi R^2 n$

$$\begin{aligned}\Delta A &= 4\pi\left[nr^2 - R^2\right] \\ &= 4\pi\left[\frac{R^3}{r^3}r^2 - R^2\right] \\ &= 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]\end{aligned}$$

Energy released $W = T \times \Delta T$

$= T \times 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$ ..... (i)

Heat produced $Q = \frac{W}{J}$ ..... (ii)

$Q = m \cdot s \cdot \Delta\theta$

Put (i) and (iii) into (ii)

$$m \cdot S\Delta\theta = \frac{4\pi R^3 T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$
$$\frac{4}{3}\pi R\rho_{water} \quad S_{water} \quad \Delta\theta = \frac{4\pi R^3 T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$
$$\Delta Q = \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

Question 117

In resonance tube, first and second resonance are obtained at depths 22.7 cm and 70.2 cm respectively. The third resonance will be obtained at a depth

Options:

A. 117.7 cm

B. 92.9 cm

C. 115.5 cm

D. 113.5 cm

Answer: A

Solution:

First resonance will occur at $l_1 + x = \frac{\lambda}{4}$ .... (i)

Second resonance will occur at $l_2 + x = \frac{3\lambda}{4}$ ..... (ii)

$$l_2 + x = 3(l_1 + x)$$
$$l_2 + x = 3l_1 + 3x$$
$$2x = l_2 - 3l_1$$
$$\therefore \quad x = \frac{l_2 - 3l_1}{2}$$
$$= \frac{70.2 - 68.1}{2} = 1.05 \text{ cm}$$

$\therefore$ Third resonance occurs at $l_3 + x = \frac{5\lambda}{4}$

$\therefore \quad l_3 = 5(l_1 + x) - x$
$= 5l_1 + 4x$
$= 113.5 + 4.2$
$= 117.7 \text{ cm}$

Question 118

Twenty seven droplets of water each of radius 0.1 mm merge to form a single drop then the energy released is

Options:

A. 1.6×10^{-3} J

B. 1.6 J

C. 1600 J

D. 1.6×10^{-7} J

Answer: D

Solution:

From

$$W = 4\pi r^2 T \left(n - n^{2/3}\right)$$

$$W = 4\pi \times \left(0.1 \times 10^{-3}\right)^2 \times 0.072 \left[27 - (3)^{2/3}\right]$$
$$= 1.627 \times 10^{-7} \quad \left(T_{\text{water}} \quad = 0.075 \text{ N/m}\right)$$

Question 119

If two inputs of a NAND gate are shorted, the resulting gate is

Options:

A. an OR gate

B. an AND gate

C. a NOT gate

D. a NOR gate

Answer: C

Solution:

When the two inputs of a NAND gate are shorted, the resulting gate behaves as a NOT gate. This is because a NAND gate outputs the opposite of the AND operation. When both inputs are the same (due to shorting), the NAND gate essentially inverts that single input signal.

To illustrate :

- If the input is 0 (0 AND 0), a regular AND gate would output 0, but a NAND gate outputs the opposite, which is 1.
- If the input is 1 (1 AND 1), a regular AND gate would output 1, but a NAND gate outputs the opposite, which is 0.

So, with both inputs shorted, the NAND gate effectively becomes a NOT gate, inverting whatever single input it receives.

Therefore, the correct answer is :

Option C : a NOT gate.

Question 120

Venturimeter is used to

Options:

A. measure liquid pressure.

B. measure liquid density.

C. measure rate of flow of liquids.

D. measure surface tension.

Answer: C

Solution:

A Venturi meter measures liquid flow in a pipe using a narrow section and works on the basis of Bernoulli's principle.

Question 121

Which of the following statements is 'WRONG' for the conductors?

Options:

A. In static situation, the interior of conductor can have no charge.

B. The net electrostatic field is zero in the interior of a conductor.

C. The electrostatic field just outside the surface of charged conductor must be tangential to the surface at any point.

D. The electrostatic potential is constant within and on the surface of a conductor.

Answer: C

Solution:

The electric field just outside the surface of the conductor is perpendicular to the surface which is a property of conductors in electrostatic equilibrium.

Question 122

A circular coil of radius 'r' and number of turns ' n ' carries a current 'I'. The magnetic fields at a small distance 'h' along the axis of the coil (B_a) and at the centre of the coil $(\mathrm{B_c})$ are measured. The relation between B_c and B_a is

Options:

A. $\mathrm{B_c} = \mathrm{B_a}\left(1 + \frac{\mathrm{h}^2}{\mathrm{r}^2}\right)$

B. $B_c = B_a\left(1+\frac{h^2}{r^2}\right)^{\frac{1}{2}}$

C. $B_c = B_a\left(1+\frac{h^2}{r^2}\right)^{\frac{3}{2}}$

D. $B_c = B_a\left(1+\frac{h^2}{r^2}\right)^{-\frac{3}{2}}$

Answer: C

Solution:

Magnetic field along the axis of the coil is:

$B_a = \frac{\mu_0}{4\pi}\left[\frac{2\pi n I r^2}{(r^2+h^2)^{\frac{3}{2}}}\right]$ .... (i)

Magnetic field at the centre of the coil is:

$$B_C = \frac{\mu_0}{4\pi}\left(\frac{2\pi n I}{r}\right) \quad .... \text{(ii)}$$

$$\frac{B_C}{B_a} = \frac{(r^2+h^2)^{\frac{3}{2}}}{r^3}$$

$$\therefore \quad B_C = B_a\left[1+\frac{h^2}{r^2}\right]^{\frac{3}{2}}$$

Question 123

A spherical surface of radius of curvature 'R' separates air from glass of refractive index 1.5. The centre of curvature is in the glass. A point object P placed in air forms a real image Q in the glass. The line PQ cuts the surface at point 'O' and $PO = OQ = x$. Hence the distance 'x' is equal to

Options:

A. 1.5 R

B. 2 R

C. 3 R

D. 5 R

Answer: D

Solution:

Given: $u = -x, v = +x$

We know,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$
$$\Rightarrow \frac{1.5}{x} + \frac{1}{x} = \frac{0.5}{R}$$
$$\frac{2.5}{x} = \frac{0.5}{R}$$
$$\therefore \quad x = 5R$$

Question 124

The rotational kinetic energy and translational kinetic energy of a rolling body are same, the body is

Options:

A. disc

B. sphere

C. cylinder

D. ring

Answer: D

Solution:

Translational kinetic energy of a ring is:

$KE_{Trans} \quad = \frac{1}{2}mv^2$

Rotational kinetic energy of the ring is:

$$\begin{aligned} KE_{Rolling} &= \frac{1}{2}I^2 \\ &= \frac{1}{2}mR^2\omega^2 \\ KE_{Rolling} &= \frac{1}{2}\,mv^2 \quad (\because v = \omega R) \end{aligned}$$

Question 125

Two concentric circular coils A and B have radii 20 cm and 10 cm respectively lie in the same plane. The current in coil A is 0.5 A in anticlockwise direction. The current in coil B so that net field at the common centre is zero, is

Options:

A. 0.5 A in anticlockwise direction

B. 0.25 A in anticlockwise direction.

C. 0.25 A in clockwise direction.

D. 0.125 A in clockwise direction.

Answer: C

Solution:

Magnetic field at the centre of a circular loop

$B = \frac{\mu_0 NI}{2R}$

Net Magnetic field $B_{net} = 0$ (given)

$$\begin{aligned} \Rightarrow B_{net} &= \frac{\mu_0 N_1 \times 0.5}{2 \times (0 \cdot 2)} - \frac{\mu_0 N_1 \times x}{2 \times (0 \cdot 1)} \\ \Rightarrow \frac{\mu_0 N_1 x}{0.2} &= \frac{\mu_0 N_1 0.5}{0.4} \\ \therefore \quad x &= \frac{0.5 \times 0.2}{0.4} \\ &= 0.25 \text{ A in clockwise direction} \end{aligned}$$

Question 126

A charged spherical conductor of radius 'R' is connected momentarily to another uncharged spherical conductor of radius 'r' by means of a thin conducting wire, then the ratio of the surface charge density of the first to the second conductor is

Options:

A. R : r^2

B. R : r

C. r : R

D. 1 : 1

Answer: C

Solution:

$V_1 = V_2$ ($\because$ both the conductors have the same potential as they are connected)

$$\Rightarrow \frac{q_1}{R} = \frac{q_2}{r}$$

$$\sigma = \frac{q}{4\pi R^2}$$

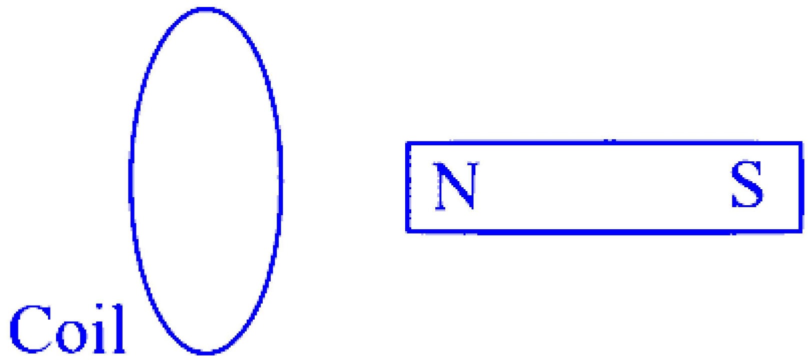
$\therefore$ The ratio of their charge densities is:

$$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{\frac{q_1}{R^2}}{\frac{q_2}{r^2}}$$

$$\therefore \frac{\sigma_1}{\sigma_2} = \frac{r}{R}$$

Question 127

The magnet is moved towards the coil with speed 'V'. The induced e.m.f. in the coil is 'e'. The magnet and the coil move away from one another each moving with speed 'V'. The induced e.m.f. in the coil is



Options:

A. e

B. 2e

C. $\frac{e}{2}$

D. 4e

Answer: B

Solution:

The equation for the induced emf is:

$e = B/V$

Relative velocity between the coil and the magnet is:

$v_r = 2v$

$\therefore$ The new induced emf in the coil is:

$e_{new} = Bl \cdot 2V = 2e$

Question 128

A uniform wire $20 \mathrm{~m}$ long and weighing $50 \mathrm{~N}$ hangs vertically. The speed of the wave at mid point of the wire is (acceleration due to gravity $= g = 10 \mathrm{~ms}^{-2}$)

Options:

A. 4 ms^{-1}

B. $10\sqrt{2} \text{ ms}^{-1}$

C. 10 ms^{-1}

D. Zero ms^{-1}

Answer: C

Solution:

$m = \frac{50}{10} = 5 \text{ kg} \quad (\because W = mg)$

Tension in the mid-point of the wire is:

$T = \frac{m}{2} g = \frac{5}{2} \times 10 = 25 \text{ N}$

$\therefore$ Speed of the wave at mid-point of the wire is:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{25}{\left(\frac{5}{20}\right)}} \quad \left(\because \mu = \frac{m}{L}\right)$$

$\therefore \quad v = 10 \text{ m/s}$

Question 129

A large number of bullets are fired in all directions with same speed 'U'. The maximum area on the ground on which the bullets will spread is

Options:

A. $\frac{\pi u^2}{g}$

B. $\frac{\pi u^4}{g^2}$

C. $\frac{\pi^2 u^4}{g^2}$

D. $\frac{\pi^2 u^2}{g^2}$

Answer: B

Solution:

Area in which bullet will spread $= \pi r^2$

For maximum area, $r = R_{max} = \frac{u^2}{g}$ [when $\theta = 45°$]

Maximum area $\pi R_{max}^2 = \pi\left(\frac{u^2}{g}\right)^2 = \frac{\pi u^4}{g^2}$

Question 130

A transformer has 20 turns in the primary and 100 turns in the secondary coil. An ac voltage of $V_{in} = 600 \sin 314t$ is applied to primary terminal of transformer. Then maximum value of secondary output voltage obtained in volt is

Options:

A. 600

B. 300

C. 3000

D. 6000

Answer: C

Solution:

We know,

$\frac{V_s}{V_p} = \frac{N_s}{N_p}$

$\therefore$ The maximum value of secondary output voltage is:

$$V_s = \frac{N_s}{N_p} \times V_p = \frac{100}{20} \times 600$$
$$V_s = 3000V$$

Question 131

On increasing the reverse bias to a large value in a P-N junction diode, current

Options:

A. increase slowly.

B. remains fixed.

C. suddenly increases.

D. decreases slowly.

Answer: C

Solution:

When the reverse voltage increases beyond the breakdown voltage, the current suddenly increases.

Question 132

For an ideal gas the density of the gas is ρ_0 when temperature and pressure of the gas are T_0 and P_0 respectively. When the temperature of the gas is $2\ \mathrm{T}_0$, its pressure will be $3\mathrm{P}_0$. The new density will be

Options:

A. $\frac{3}{2}\rho_0$

B. $\frac{4}{3}\rho_0$

C. $\frac{3}{4}\rho_0$

D. $\frac{2}{3}\rho_0$

Answer: A

Solution:

Density $\propto P/T$

So, $\frac{d_2}{d_1} = \frac{P_2}{T_2} \times \frac{T_1}{P_1}$

$\therefore$ The new density is:

$$d_2 = \rho_0 \times \frac{3P_0}{2\,T_0} \times \frac{T_0}{P_0}$$
$$d_2 = \frac{3}{2}\rho_0$$

Question 133

The temperature gradient in a rod of length 75 cm is $40^\circ C/m$. If the temperature of cooler end of the rod is $10^\circ C$, then the temperature of hotter end is

Options:

A. $50^\circ C$

B. $40^\circ C$

C. $35^\circ C$

D. $25^\circ C$

Answer: B

Solution:

We know

$$T_g = \frac{T_1 - T_2}{x}$$
$$\Rightarrow \frac{T_1 - 10}{0.75} = 40$$

$\therefore$ Temperature of the hotter end is:

$$T_1 = (40 \times 0.75) + 10$$
$$T_1 = 40^\circ C$$

Question 134

In an oscillating LC circuit, the maximum charge on the capacitor is 'Q'. When the energy is stored equally between the electric and magnetic fields, the charge on the capacitor becomes

Options:

A. $\frac{Q}{4}$

B. $\frac{Q}{2}$

C. $\frac{Q}{\sqrt{2}}$

D. $\frac{Q}{\sqrt{3}}$

Answer: C

Solution:

Maximum energy stored in a capacitor, $E_1 = \frac{Q^2}{2C}$

When energy is stored equally between the electric and magnetic fields, then energy in the capacitor is $E_2 = \frac{1}{2}E_1$

If Q' is the charge on the capacitor in this case, then $E_2 = \frac{Q'^2}{2C}$.

$$\therefore \quad \frac{Q'^2}{2C} = \frac{1}{2}\frac{Q^2}{2C}$$

$$Q' = \frac{Q}{\sqrt{2}}.$$

Question 135

A body of mass 'm' is raised through a height above the earth's surface so that the increase in potential energy is $\frac{mgR}{5}$. The height to which the body is raised is (R = radius of earth, g = acceleration due to gravity)

Options:

A. R

B. $\frac{R}{2}$

C. $\frac{R}{4}$

D. $\frac{R}{8}$

Answer: C

Solution:

When a particle of mass m is taken from the Earth's surface to a height $h = nR$, then the change in P.E. can be calculated as,

$$\Delta U = mgR\left(\frac{n}{n+1}\right)$$
$$\therefore \quad \frac{mgR}{5} = mgR\left(\frac{n}{n+1}\right)$$
$$\therefore \quad n + 1 = 5n$$
$$\therefore \quad n = \frac{1}{4}$$
$$\therefore \quad h = \frac{R}{4}$$

Question 136

With increase in frequency of a.c. supply, the impedance of an L-C-R series circuit

Options:

A. remains constant.

B. increases.

C. decreases.

D. decreases at first, becomes minimum and then increases.

Answer: D

Solution:

We know

$X_L = L\omega$ and $X_C = \frac{1}{C\omega}$

$\Rightarrow$ When the frequency increases, X_L increases and X_C decreases.

$\therefore$ The impedance of an LCR series circuit decreases at first, becomes minimum and then increases.

Question 137

A passenger is sitting in a train which is moving fast. The engine of the train blows a whistle of frequency 'n'. If the apparent frequency of sound heard by the passenger is 'f' then

Options:

A. $f = n$

B. $f > n$

C. $f < n$

D. $f \leq n$

Answer: A

Solution:

The situation described involves the Doppler effect, but with a key distinction: both the source of the sound (the train's engine) and the observer (the passenger) are moving at the same speed in the same direction since they are both on the train.

In the Doppler effect, the frequency of a wave changes due to the relative motion between the source and the observer. However, if the source and the observer are moving together at the same velocity (as is the case here with the passenger and the train's engine), there is no relative motion between them. Consequently, the frequency of the sound heard by the observer (passenger) will be the same as the actual frequency of the sound emitted by the source (train's engine).

Therefore, the correct answer is :

Option A : $f = n$

Question 138

If the magnitude of intensity of electric field at a distance 'r_1' on an axial line and at a distance 'r_2' on an equatorial line due to a given short dipole are equal, then $r_1 : r_2$, is

Options:

A. $\sqrt[3]{2} : 1$

B. $\sqrt{2} : 1$

C. $1 : 2$

D. $1 : \sqrt[3]{2}$

Answer: A

Solution:

$$E_{\text{axial}} = E_{\text{equatorial}} \quad \Rightarrow k\frac{2p}{r_1^3} = \frac{kp}{r_2^3}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{2^{1/3}}{1} = \sqrt[3]{2} : 1$$

Question 139

In an a.c. circuit the instantaneous current and emf are represented as $\mathrm{I} = \mathrm{I}_0, \sin[\omega t - \pi/6]$ and $\mathrm{E} = \mathrm{E}_0 \sin[\omega t + \pi/3]$ respectively. The voltage leads the current by

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

Answer: A

Solution:

Given $\phi_1 = \frac{\pi}{6}$ and $\phi_2 = \frac{\pi}{3}$

$\therefore \quad \Delta\phi = \frac{\pi}{3} - \left(\frac{-\pi}{6}\right) = \frac{\pi}{2}$

Question 140

In a biprism experiment, monochromatic light of wavelength 'λ' is used. The distance between two coherent sources 'd' is kept constant. If the distance between slit and eyepiece 'D' is varied as $D_1, D_2, D_3 \& D_4$ and corresponding measured fringe widths are Z_1, Z_2, Z_3 and Z_4 then

Options:

A. $\mathrm{Z_1D_1 = Z_2D_2 = Z_3D_3 = Z_4D_4}$

B. $\frac{Z_1}{D_1} = \frac{Z_2}{D_2} = \frac{Z_3}{D_3} = \frac{Z_4}{D_4}$

C. $\mathrm{D_1\sqrt{Z_1} = D_2\sqrt{Z_2} = D_3\sqrt{Z_3} = D_4\sqrt{Z_4}}$

D. $Z_1\sqrt{D_1} = Z_2\sqrt{D_2} = Z_3\sqrt{D_3} = Z_4\sqrt{D_4}$

Answer: B

Solution:

Fringe width $Z = \frac{\lambda D}{d}$

$\therefore \quad \frac{Z}{D} = \frac{\lambda}{d} =$ constant, as λ and d are constant

$\therefore \quad \frac{Z_1}{D_1} = \frac{Z_2}{D_2} = \frac{Z_3}{D_3} = \frac{Z_4}{D_4}$

Question 141

Three charges each of value $+q$ are placed at the corners of an isosceles triangle ABC of sides AB and AC each equal to $2\mathrm{a}$. The mid

points of AB and AC are D and E respectively. The work done in taking a charge Q from D to E is ($\varepsilon_0 =$ permittivity of free space)

Options:

A. Zero

B. $\frac{3qQ}{4\pi\varepsilon_0 a}$

C. $\frac{qQ}{8\pi\varepsilon_0 a}$

D. $\frac{3qQ}{8\pi\varepsilon_0 a}$

Answer: A

Solution:

Given $AB = AC = 2a$

$V_D = V_E$ ($\because$ D and E are mid-points)

$\therefore$ The work done in taking a charge q from D to E is

$W = q\Delta V = q(V_D - V_E)$

$\therefore$ $W = 0$ (Equipotential surfaces)

Question 142

Light of frequency 1.5 times the threshold frequency is incident on photosensitive material. If the frequency is halved and intensity is doubled, the photocurrent becomes

Options:

A. quadrupled

B. double

C. half

D. zero

Answer: D

Solution:

If threshold frequency is ν_0, then light frequency becomes 1.5 ν_0.

If we make it half it becomes 0.75 ν_0, which is smaller than threshold frequency, therefore photoelectric current is zero.

Question 143

A solid cylinder of mass 3 kg is rolling on a horizontal surface with velocity 4 m/s. It collides with a horizontal spring whose one end is fixed to rigid support. The force constant of material of spring is 200 N/m. The maximum compression produced in the spring will be (assume collision between cylinder & spring be elastic)

Options:

A. 0.7 m

B. 0.2 m

C. 0.5 m

D. 0.6 m

Answer: D

Solution:

At maximum compression, the solid cylinder will stop.

So loss in K.E. of cylinder = Gain in P.E. of spring

$$\therefore \quad \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}kx^2$$
$$\therefore \quad \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\left(\frac{v}{R}\right)^2 = \frac{1}{2}kx^2$$
$$\therefore \quad \frac{3}{4}mv^2 = \frac{1}{2}kx^2$$
$$\therefore \quad \frac{3}{4} \times 3 \times (4)^2 = \frac{1}{2} \times 200 \times x^2$$
$$\therefore \quad \frac{36}{100} = x^2 \Rightarrow x = 0.6 \text{ m}$$

Question 144

Which one of the following statements is Wrong?

Options:

A. A body can have zero velocity and still be accelerated.

B. A body can have a constant velocity and still have a varying speed.

C. A body can have a constant speed and still have a varying velocity.

D. The direction of the velocity of a body can change when its acceleration is constant.

Answer: B

Solution:

Let's analyze each statement to determine which one is wrong :

Option A : A body can have zero velocity and still be accelerated.

- This statement is true. For example, at the highest point of its trajectory, a thrown ball has zero velocity, but it is still accelerating due to gravity.

Option B : A body can have a constant velocity and still have a varying speed.

- This statement is false. Velocity is a vector quantity that includes both speed and direction. If the velocity is constant, it means both the speed and the direction of the body are constant. Therefore, a body with constant velocity cannot have a varying speed.

Option C : A body can have a constant speed and still have a varying velocity.

- This statement is true. A body moving in a circular path at a constant speed has a constantly changing velocity because the direction of the velocity vector changes even though its magnitude (speed) remains constant.

Option D : The direction of the velocity of a body can change when its acceleration is constant.

- This statement is true. A common example is an object in free fall under gravity. The direction of its velocity changes as it goes up, stops, and comes back down, even though the acceleration (due to gravity) is constant.

Therefore, the wrong statement is Option B : A body can have a constant velocity and still have a varying speed.

Question 145

The fundamental frequency of a sonometer wir carrying a block of mass 'M' and density 'ρ' is 'n' Hz. When the block is completely immerse in a liquid of density 'σ' then the new frequency will be

Options:

A. $\mathrm{n}\left[\frac{\rho-\sigma}{\rho}\right]^{\frac{1}{2}}$

B. $\mathrm{n}\left[\frac{\rho-\sigma}{\sigma}\right]^{\frac{1}{2}}$

C. $\mathrm{n}\left[\frac{\rho}{\rho-\sigma}\right]^{\frac{1}{2}}$

D. $n\left[\frac{\sigma}{\rho-\sigma}\right]^{\frac{1}{2}}$

Answer: A

Solution:

$$\begin{aligned} & \mathrm{n} \propto \sqrt{\mathrm{T}} \\ & \mathrm{T}=\mathrm{mg}=\rho \mathrm{Vg} \\ \therefore \quad & \mathrm{T} \propto \sqrt{\rho \mathrm{Vg}} \end{aligned}$$

After immersion in the liquid,

$$\therefore \quad \frac{\mathrm{n}_2}{\mathrm{n}} \propto \frac{\sqrt{\mathrm{V}(\rho-\sigma) \mathrm{g}}}{\sqrt{\mathrm{V} \rho \mathrm{g}}}$$

$$\therefore \quad \mathrm{n}_2=\mathrm{n}\left[\frac{\rho-\sigma}{\rho}\right]^{\frac{1}{2}}$$

Question 146

A simple pendulum has a time period 'T' in air. Its time period when it is completely immersed in a liquid of density one eighth the density of the material of bob is

Options:

A. $\left(\sqrt{\frac{7}{8}}\right)\mathrm{T}$

B. $\left(\sqrt{\frac{5}{8}}\right)\mathrm{T}$

C. $\left(\sqrt{\frac{3}{8}}\right)\mathrm{T}$

D. $\left(\sqrt{\frac{8}{7}}\right)\mathrm{T}$

Answer: D

Solution:

Time period of simple pendulum $\mathrm{T}=2\pi\sqrt{\frac{l}{\mathrm{g}}}$

$\Rightarrow \mathrm{T}\propto\frac{1}{\sqrt{\mathrm{g}}}$

Net downward force acting on the bob inside the liquid = Weight of bob − Upthrust

$\Rightarrow \mathrm{Vpg}-\mathrm{V}\frac{\rho}{\rho}\mathrm{g}=\frac{7}{8}\,\mathrm{V}\rho\mathrm{g}$

The value of g inside the liquid will be $\frac{g}{8}$

$\therefore$ Time period in liquid $\mathrm{T}_1=\frac{1}{2\pi}\sqrt{\frac{l}{\frac{7}{8}}}=\sqrt{\frac{8}{7}}\,\mathrm{T}$

Question 147

Array of light is incident at an angle of incidence 'i' on one surface of a prism of small angle A and emerges normally from the other surface. If the refractive index of the material of the prism is 'μ', then the angle of incidence is equal to

Options:

A. $\frac{\mathrm{A}}{2\mu}$

B. $\frac{A\mu}{2}$

C. $A\mu$

D. $\frac{A}{\mu}$

Answer: C

Solution:

Given: $e = 0$

$\therefore \quad r_2 = 0, \ A = r_1$

Since 'i' is small, Snell's law of refraction can be modified to,

$$\mu = \frac{i}{r_1}$$

$$\therefore \quad i = \mu r_1 = \mu A$$

Question 148

An isotope of the original nucleus can be formed in a radioactive decay, with the emission of following particles.

Options:

A. one α and one β

B. one α and two β

C. one α and four β

D. four α and one β

Answer: B

Solution:

$${}_{Z}X^{A} \xrightarrow{\alpha} {}_{Z-2}Y^{A-4} \xrightarrow{2\beta^{-}} {}_{Z}X^{A-4}$$

Question 149

If two identical spherical bodies of same material and dimensions are kept in contact, the gravitational force between them is proportional to R^X, where x is non zero integer [Given : R is radius of each spherical body]

Options:

A. -4

B. 4

C. 2

D. -2

Answer: B

Solution:

$$F = \frac{G \times m \times m}{(2R)^2} = \frac{G \times \left(\frac{4}{3}\pi R^3 \rho\right)^2}{4R^2}$$
$$= \frac{4}{9}\pi^2 \rho^2 R^4$$
$$\therefore \quad F \propto R^4$$

Question 150

A and B are two interfering sources where A is ahead in phase by 54° relative to B. The observation is taken from point P such that PB $-$ PA = 2.5 λ. Then the phase difference between the waves from A and B reaching point P is (in rad)

Options:

A. 3.5π

B. 5.3π

C. 4.3π

D. 5.8π

Answer: B

Solution:

$$
\begin{aligned}
&\text{Total phase difference } = \phi_1 + \phi_2 \\
&\quad \phi_1 = 54 \times \frac{\pi}{180} = 0.3\pi \\
&\quad \phi_2 = \frac{2\pi}{\lambda} \times (\mathrm{PB} - \mathrm{PA}) \\
&\quad\quad = \frac{2\pi}{\lambda} \times 2.5\lambda = 5\pi \\
&\therefore \quad \phi_1 + \phi_2 = 5\pi + 0.3\pi = 5.3\pi
\end{aligned}
$$